IFAC

International Federation of Automatic Control

BENCHMARK PROBLEMS FOR CONTROL SYSTEM DESIGN

REPORT OF THE IFAC THEORY COMMITTEE

Edited by **Edward J. Davison**, Chairman

May 1990

Benchmark Problems for Control System Design

Abstract

This report contains a collection of potential "benchmark problems" for the Control Community and has been completed by the IFAC Theory Committee over the period July 1987 - July 1990. The motivation for carrying out such a study is that it was believed by the IFAC Theory Committee that it would be useful to have a collection of standard problems for comparing the benefits of "new or existing control system design tools", i.e. at present, every new design method is applied to some ad hoc example, and it is difficult to determine a meaningful comparison between existing techniques.

The procedure for collecting such benchmark problems was as follows. A formal request to the Control Community asking for potential benchmark problems appeared in:

(a) The IFAC Newsletter No. 5, October 1988.

(b) The IEEE Control System Society Magazine, February 1989;

and in addition:

(c) A request for submission of potential problems from the IFAC Theory Committee members (representing 32 countries) was made.

i.e. potential problems were sought from as wide an audience as possible. The submitted problems were then reviewed, and a subset of the problems selected, with a view of obtaining the widest possible selection of representative "real world" problems. (Page limitations dictated that only a subset of submitted problems could be accepted.)

With the exception of one problem (90-13), all of the enclosed benchmark problems are modelled from the "real world"; the majority of the problems are multivariable, and many of the problems have associated with them structured uncertainties, as well as constraints of various types associated with either inputs or outputs of the system. All of the problems have associated with them a contact address, where additional information on the problem may be obtained on request.

Edward J. Davison

May 1990

List of Benchmark Problems

List of Contributors to Report

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Characterization of Benchmark Problems by Area

Overview Description of Benchmark Problems

Problem #	Plant type	Plant order	No. of inputs	No. of outputs	No. of measurable outputs	No. of explicit disturbances	No. of uncertain parameters	Characteristics of problem
90-01 Dis. Column	L,C	11	3	3	unspecified	1	-	constraints imposed
90-02 Boiler	L,C	9	3	2	2	1	-	non-minimum phase plant
90-03 Shell	L,C	time delay	3	2	7	2	3	constraints imposed, plant parameter variations specified
90-04 Rolling Mill	L,D	10	3	5	5	3	-	constraints imposed
90-05 Missile	L,C	3	1	1	1	-	2	constraints imposed, plant parameter variations specified
90-06 High Order Plant	L,C	55	2	2	2	-	-	non-minimum phase plant, constraints imposed
90-07 Hydraulic System	L,C	3	1	1	1	1	8	constraints imposed, plant parameter variations specified
90-08 Underwater Vehicle	L,C	8	2	1	2	2	1	constraints imposed, decentralized control structure imposed, plant parameter variations specified
90-09 Pendula	L,C	2 to 20	1 to 10	1 to 10	1 to 10	-	-	non-minimum phase plant
90-10 Ship Heading	L,C	3	1	1	1	-	-	constriants imposed
90-11 Bus	L,C	5	1	1	4	-	2	constraints imposed, plant parameter variations specified
90-12 Robotics	NL,C	12	3	3	3	-	-	3 degree of freedom + DC motor dynamics
90-13 Robust	NL,C	n	1	1	1	-	4	constraints imposed

L = linear plant, C = continuous time model
NL = nonlinear plant, D = discrete time model

Acknowledgement

On behalf of the IFAC Community, I wish to give thanks to the following members of the IFAC Theory Committee:

J. Ackermann	Vice-Chairman
A. Benveniste	Vice-Chairman
Han-Fu Chen	Vice-Chairman
M. Grimble	Vice-Chairman
G. Goodwin	Vice-Chairman
Y. Tsypkin	Vice-Chairman

who carried out the difficult process of reviewing all submitted potential benchmark problems for the report, and to Linda Espeut for her outstanding work in organizing and typing of the report.

Edward J. Davison, Chairman

IFAC Theory Committee 1987-1990

Note of Disclaimer

IFAC bears no responsibility re the accuracy or validity of the enclosed Benchmark problems.

Process Control

Problem #90-01 (Binary Distillation Column)
Problem #90-02 (Drum Boiler Problem)
Problem #90-03 (Shell Control Problem)
Problem #90-04 (A Cold Rolling Mill)

Problem #90-01 BINARY DISTILLATION COLUMN

(A) General Description

This problem describes a fairly realistic model of a binary distillation column, and has the feature that pressure variation is included in the model's description; the system is multivariable, with 3 inputs and 3 outputs, and includes one disturbance input.

(B) Reference

The formulation of this problem is given in:

Davison, E.J., "Control of a distillation column with pressure variation", *Trans. Institute of Chemical Engineers*, vol. 45, 1967, pp. T229-250.

(C) Problem Description

The equation of a binary distillation column with n plates for a general binary system of components is given in the above reference. The following linearized model is obtained from the above reference, for a column containing 8 plates:

$$\begin{aligned} \dot{x} &= Ax + Bu + E\omega \\ y &= Cx \\ y_m &= x \end{aligned}$$

where $u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$, $y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$, $\omega = \omega_1$, where:

x_1 = composition of more volatile component in condenser
x_{10} = composition of more volatile component in reboiler
x_{11} = pressure
x_2 = composition of more volatile component in plate # 1
⋮
x_9 = composition of more volatile component in plate #8
y_1 = composition of more volatile component in bottom product
y_2 = composition of more volatile component in top product
y_3 = pressure

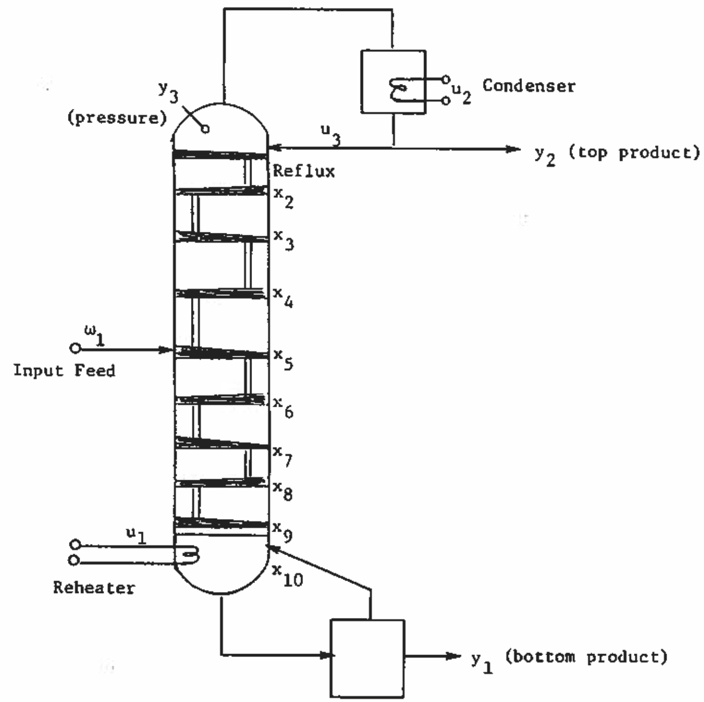

Figure 1: Binary distillation column with pressure variation

```
A =

  Columns 1 through 6

 -1.4000e-02   4.3000e-03            0            0            0            0
  9.5000e-03  -1.3800e-02   4.6000e-03            0            0            0
           0   9.5000e-03  -1.4100e-02   6.3000e-03            0            0
           0            0   9.5000e-03  -1.5800e-02   1.1000e-02            0
           0            0            0   9.5000e-03  -3.1200e-02   1.5000e-02
           0            0            0            0   2.0200e-02  -3.5200e-02
           0            0            0            0            0   2.0200e-02
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
  2.5500e-02            0            0            0            0            0

  Columns 7 through 11

           0            0            0            0            0
           0            0            0            0   5.0000e-04
           0            0            0            0   2.0000e-04
           0            0            0            0            0
           0            0            0            0            0
  2.2000e-02            0            0            0            0
 -4.2200e-02   2.8000e-02            0            0            0
  2.0200e-02  -4.8200e-02   3.7000e-02            0   2.0000e-04
           0   2.0200e-02  -5.7200e-02   4.2000e-02   5.0000e-04
           0            0   2.0200e-02  -4.8300e-02   5.0000e-04
           0            0            0   2.5500e-02  -1.8500e-02
```

```
B =

           0            0            0
  5.0000e-06  -4.0000e-05   2.5000e-03
  2.0000e-06  -2.0000e-05   5.0000e-03
  1.0000e-06  -1.0000e-05   5.0000e-03
           0            0   5.0000e-03
           0            0   5.0000e-03
 -5.0000e-06   1.0000e-05   5.0000e-03
 -1.0000e-05   3.0000e-05   5.0000e-03
 -4.0000e-05   5.0000e-06   2.5000e-03
 -2.0000e-05   2.0000e-06   2.5000e-03
  4.6000e-04   4.6000e-04            0
```

```
E =

           0
           0
           0
           0
  1.0000e-02
           0
           0
           0
           0
           0
           0
```

```
C =

     0     0     0     0     0     0     0     0     0     1     0
     1     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     1
```

u_1 = reboiler steam temperature

u_2 = condenser coolant temperature

u_3 = controlled reflux

ω_1 = change of input feed concentration

y_m = outputs of the system which can be measured

It is desired to design a controller to regulate the 3 outputs y_1, y_2, y_3 against the unmeasurable disturbance ω_1 (and other unmeasurable disturbances), with as fast a settling time as possible, subject to the constraint:

$$\left.\begin{array}{l} |u_1| \le 2.5 \\ |u_2| \le 2.5 \\ |u_3| \le 0.30 \end{array}\right\}, t \ge 0$$

for $|\omega_1| \le 1$, $t \ge 0$. In this problem, the choice of what measurable outputs to use in the controller, is considered to be part of the problem statement; in general, a controller which uses the fewest number of output measurements is desirable.

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Problem #90-02 DRUM BOILER PROBLEM

(A) General Description

This problem describes a fairly realistic model of a drum boiler and has the feature of being multivariable, unstable, and non-minimum phase.

(B) Reference

The source of this problem is given in:

Chien, K.L., Ergin, E.I., Ling, C., Lee, A., "Dynamic analysis of a boiler", *Trans. ASME*, vol. 80, 1958, pp. 1809-1819.

(C) Problem Description

The following plant

$$\begin{aligned} \dot{x} &= Ax + Bu + E\omega \\ y &= Cx \end{aligned}$$

is a linearized model of a drum boiler where $u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$, $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$, $\omega = \omega_1$, $x \in R^9$,

where:

x_1 = density of output steam flow
x_2 = temperature of output steam flow
x_3 = temperature of superheater
x_4 = quality of steam
x_5 = water flow in riser
x_6 = pressure
x_7 = temperature of riser
x_8 = temperature of water in boiler
x_9 = water level in boiler
y_1 = pressure
y_2 = water level in boiler
u_1 = input fuel flow

A =

Columns 1 through 6

-3.9300e+00	-3.1500e-03	0	0	0	4.0300e-05
3.6800e+02	-3.0500e+00	3.0300e+00	0	0	-3.7700e-03
2.7400e+01	7.8700e-02	-5.9600e-02	0	0	-2.8100e-04
-6.4700e-02	-5.2000e-05	0	-2.5500e-01	-3.3500e-06	3.6000e-07
3.8500e+03	1.7300e+01	-1.2800e+01	-1.2600e+04	-2.9100e+00	-1.0500e-01
2.2400e+04	1.8000e+01	0	-3.5600e+01	-1.0400e-04	-4.1400e-01
0	0	2.3400e-03	0	0	2.2200e-04
0	0	0	-1.2700e+00	1.0000e-03	7.8600e-05
-2.2000e+00	-1.7700e-03	0	-8.4400e+00	-1.1100e-04	1.3800e-05

Columns 7 through 9

0	0	0
0	0	0
0	0	0
6.3300e-05	1.9400e-04	0
1.2700e+01	4.3100e+01	0
9.0000e+01	5.6900e+01	0
-2.0300e-01	0	0
0	-7.1700e-02	0
1.4900e-03	6.0200e-03	-1.0000e-10

B =

0	0	0
0	0	0
1.5600e+00	0	0
0	-5.1300e-06	0
8.2800e+00	-1.5000e+00	3.950e-2
0	1.7800e+00	0
2.3300e+00	0	0
0	-2.4500e-02	2.840e-3
0	2.9400e-05	0

C =

0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	1

E =

-1.0000e-02
0
0
0
5.2000e+01
0
0
0
0

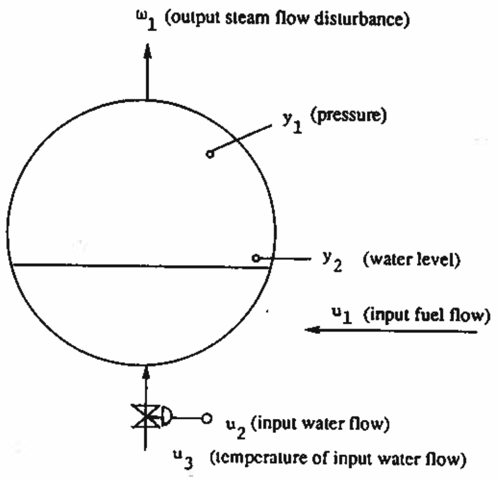

Figure 1: Drum Boiler System

u_2 = input water flow
u_3 = temperature of input water flow
ω_1 = output steam flow disturbance

It is desired to design a controller to regulate the outputs of the system against constant disturbance ω_1, and to track constant setpoints $y^{ref} \in R^2$ with as fast a settling time as possible. Note: the input u_3 is generally not used in the 'conventional control' of such a boiler.

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Problem #90-03 SHELL CONTROL PROBLEM

(A) General Description

This problem describes a representative example of a practical control problem arising in process control.

(B) Reference

The formulation of this problem is given in:

Pratt, D.M., Morari, M., ***Shell Process Control Workshop***, Buttersworth: Stoneham, MA, 1987.

(C) Problem Description

Figure 1 shows a heavy oil fractionator with three product draws and three side circulating loops. The heat requirement of the column enters with the feed, which is a gaseous stream. Product specifications for the top and side draws are determined by economics and operating requirements. There is no product specification for the bottom draw, but there is an operating constraint on the temperature in the lower part of the column. The three circulating loops remove heat to achieve the desired product separation. The heat exchangers in these loops reboil columns in other parts of the plant. Therefore, they have varying heat duty requirements. The bottom loop has an enthalpy controller which regulates heat removal in the loop by adjusting steam make. Its heat duty can be used as a manipulated variable to control the column. The heat duties of the other two loops act as disturbances to the column.

The relevant information regarding the Shell Control Problem is stated in these five subsections:

1. Control Objectives
2. Control Constraints
3. Process Model
4. Uncertainties in the Gains of the Model
5. Prototype Test Cases

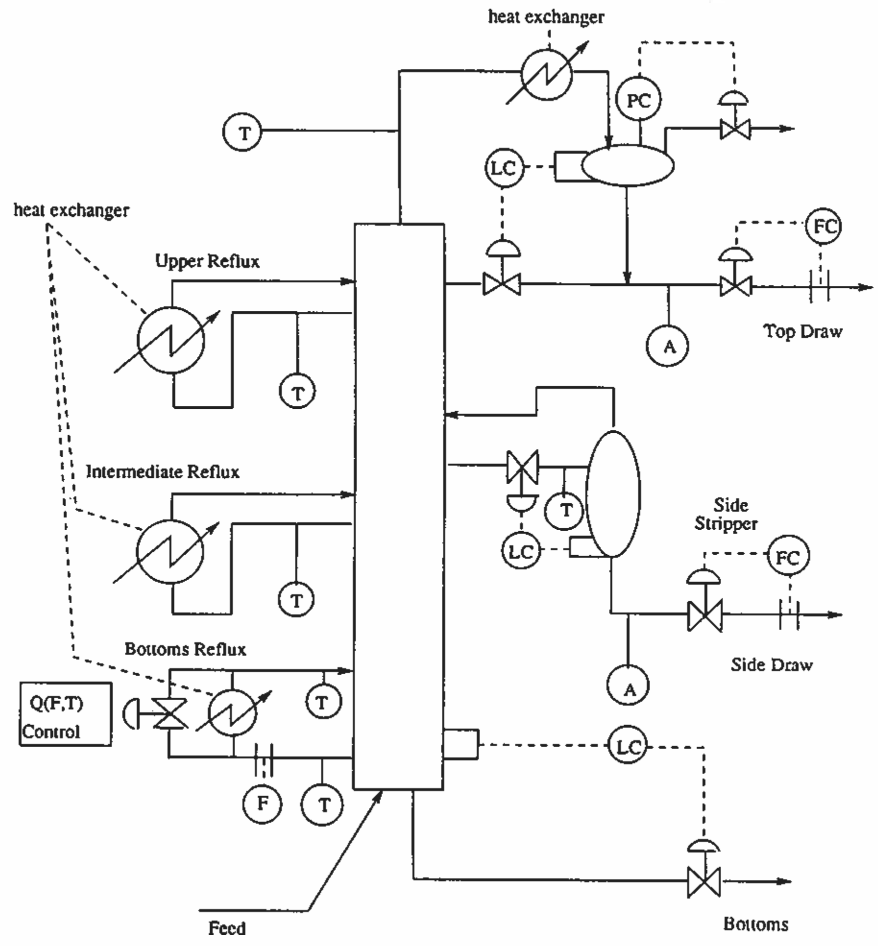

Figure 1: Diagram of a heavy oil fractionator (Shell control problem)

We have tried to encapsulate the relevant control issues in this one problem while staying as realistic as possible. The problem is stated such that an infinite number of scenarios can occur in controlling the unit. We would encourage the development of solution methodologies that are flexible enough to deal with varying (and possibly conflicting) problem requirements, and can be readily automated such that control designs can be carried out by plant personnel with only a modest knowledge of control concepts.

A complete solution to the problem should describe, in detail, the analysis and synthesis procedures that indicate that the proposed controller satisfies the control objectives for all the plants in the uncertainty set. However, because of possible discrepancies between investigators on analysis techniques, we have formulated a number of prototype test cases which form a common frame of reference for evaluating different designs.

Control Objectives

1. Maintain the top and side draw product end points at specification (0.0 ± 0.005 at steady state).

2. Maximize steam make in the steam generators (i.e., maximize heat removal) in the bottom circulating reflux. (Important note: heat duties are expressed in terms of heat input to the column. Decreasing heat duty implies increasing the amount of heat removed.)

3. Reject the unmeasured disturbances entering the column from the upper and intermediate refluxes due to change in heat duty requirements from other columns. (Upper and intermediate reflux duties range between -0.5 and 0.5.) Reject disturbances even when one or both end point analyzers fail.

4. Keep the closed-loop speed of response between 0.8 and 1.25 of the open-loop process bandwidth.

Control Constraints

1. All draws must be within hard maximum and minimum bounds of 0.5 and -0.5.

2. The bottom reflux heat duty is constrained within the hard bounds of 0.5 and -0.5.

3. All manipulated variables have maximum move size limitations of magnitude 0.05 per minute.

4. Fastest sampling time is 1 minute.

5. The bottom reflux draw temperature has a minimum value of -0.5.

6. The top endpoint must be maintained within the maximum and minimum values of 0.5 and -0.5.

Process Model (First Order Deadtime)

$$\frac{Ke^{-\theta s}}{\tau s + 1}$$

Units for θ, τ are in minutes.

		u_1 TOP DRAW	u_2 SIDE DRAW	u_3 BOTTOMS REFLUX DUTY	ω_1 INTER REFLUX DUTY	ω_2 UPPER REFLUX DUTY
y_1	TOP END POINT	$K = 4.05$ $\tau = 50$ $\theta = 27$	$K = 1.77$ $\tau = 60$ $\theta = 28$	$K = 5.88$ $\tau = 50$ $\theta = 27$	$K = 1.20$ $\tau = 45$ $\theta = 27$	$K = 1.44$ $\tau = 40$ $\theta = 27$
y_2	SIDE END POINT	$K = 5.39$ $\tau = 50$ $\theta = 18$	$K = 5.72$ $\tau = 60$ $\theta = 14$	$K = 6.90$ $\tau = 40$ $\theta = 15$	$K = 1.52$ $\tau = 25$ $\theta = 15$	$K = 1.83$ $\tau = 20$ $\theta = 15$
z_1	TOP TEMP	$K = 3.66$ $\tau = 9$ $\theta = 2$	$K = 1.65$ $\tau = 30$ $\theta = 20$	$K = 5.53$ $\tau = 40$ $\theta = 2$	$K = 1.16$ $\tau = 11$ $\theta = 0$	$K = 1.27$ $\tau = 6$ $\theta = 0$
z_3	UPPER REFLUX TEMP	$K = 5.92$ $\tau = 12$ $\theta = 11$	$K = 2.54$ $\tau = 27$ $\theta = 12$	$K = 8.10$ $\tau = 20$ $\theta = 2$	$K = 1.73$ $\tau = 5$ $\theta = 0$	$K = 1.79$ $\tau = 19$ $\theta = 0$
z_2	SIDE DRAW TEMP	$K = 4.13$ $\tau = 8$ $\theta = 5$	$K = 2.38$ $\tau = 19$ $\theta = 7$	$K = 6.23$ $\tau = 10$ $\theta = 2$	$K = 1.31$ $\tau = 2$ $\theta = 0$	$K = 1.26$ $\tau = 22$ $\theta = 0$
z_4	INTER REFLUX TEMP	$K = 4.06$ $\tau = 13$ $\theta = 8$	$K = 4.18$ $\tau = 33$ $\theta = 4$	$K = 6.53$ $\tau = 9$ $\theta = 1$	$K = 1.19$ $\tau = 19$ $\theta = 0$	$K = 1.17$ $\tau = 24$ $\theta = 0$
z_5	BOTTOMS REFLUX TEMP	$K = 4.38$ $\tau = 33$ $\theta = 20$	$K = 4.42$ $\tau = 44$ $\theta = 22$	$K = 7.20$ $\tau = 19$ $\theta = 0$	$K = 1.14$ $\tau = 27$ $\theta = 0$	$K = 1.26$ $\tau = 32$ $\theta = 0$

Uncertainties in the Gains of the Model

	TOP DRAW	SIDE DRAW	BOTTOMS REFLUX DUTY	INTER REFLUX DUTY	UPPER REFLUX DUTY
TOP END POINT	$4.05 + 2.11\varepsilon_1$	$1.77 + 0.39\varepsilon_2$	$5.88 + 0.59\varepsilon_3$	$1.20 + 0.12\varepsilon_4$	$1.44 + 0.16\varepsilon_5$
SIDE END POINT	$5.39 + 3.29\varepsilon_1$	$5.72 + 0.57\varepsilon_2$	$6.90 + 0.89\varepsilon_3$	$1.52 + 0.13\varepsilon_4$	$1.83 + 0.13\varepsilon_5$
TOP TEMP	$3.66 + 2.29\varepsilon_1$	$1.65 + 0.35\varepsilon_2$	$5.53 + 0.67\varepsilon_3$	$1.16 + 0.08\varepsilon_4$	$1.27 + 0.08\varepsilon_5$
UPPER REFLUX TEMP	$5.92 + 2.34\varepsilon_1$	$2.54 + 0.24\varepsilon_2$	$8.10 + 0.32\varepsilon_3$	$1.73 + 0.02\varepsilon_4$	$1.79 + 0.04\varepsilon_5$
SIDE DRAW TEMP	$4.13 + 1.71\varepsilon_1$	$2.38 + 0.93\varepsilon_2$	$6.23 + 0.30\varepsilon_3$	$1.31 + 0.03\varepsilon_4$	$1.26 + 0.02\varepsilon_5$
INTER REFLUX TEMP	$4.06 + 2.39\varepsilon_1$	$4.18 + 0.35\varepsilon_2$	$6.53 + 0.72\varepsilon_3$	$1.19 + 0.08\varepsilon_4$	$1.17 + 0.01\varepsilon_5$
BOTTOMS REFLUX TEMP	$4.38 + 3.11\varepsilon_1$	$4.42 + 0.73\varepsilon_2$	$7.20 + 1.33\varepsilon_3$	$1.14 + 0.18\varepsilon_4$	$1.26 + 0.18\varepsilon_5$

$$-1 \le \varepsilon_i \le 1; \quad i = 1, 2, 3, 4, 5$$

Prototype Test Cases

Demonstrate, through simulation, that the proposed controller satisfies the control objectives without violating the control constraints for the following plants within the uncertainty set (assume all inputs and outputs are initially at zero; magnitudes for the upper and intermediate reflux duty step changes are indicated below):

1. $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = \varepsilon_5 = 0$. Upper reflux duty = 0.5, intermediate reflux duty = 0.5.
2. $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = -1$; $\varepsilon_4 = \varepsilon_5 = 1$. Upper reflux duty = -0.5, intermediate reflux duty = -0.5.
3. $\varepsilon_1 = \varepsilon_3 = \varepsilon_4 = \varepsilon_5 = 1$, $\varepsilon_2 = -1$. Upper reflux duty = -0.5, intermediate reflux duty = -0.5.
4. $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = \varepsilon_5 = 1$. Upper reflux duty = 0.5, intermediate reflux duty = -0.5.
5. $\varepsilon_1 = -1$, $\varepsilon_2 = 1$, $\varepsilon_3 = \varepsilon_4 = \varepsilon_5 = 0$. Upper reflux duty = -0.5, intermediate reflux duty = -0.5.

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Problem #90-04 A COLD ROLLING MILL

(A) General Description

This problem deals with a two-stand cold rolling mill. The process dynamics are dominated by the interstand time delay. A discrete 10^{th} order model is given. The model includes disturbances, is multivariable and strongly interacting. The controller structure and controller parameters are to be found to meet realistic design specifications.

(B) Reference

A full description of the model is given in Smith, H.W., "Dynamic Control of a Two-Stand Cold Mill", *Automatica*, 5, 2, pp. 109-115, 1969, where a continuous approximation to the time delay is used. The numerical model given here has not been published previously.

(C) Problem Description

Introduction

We consider in this problem a two-stand cold rolling mill, which processes cold metal strip, previously rolled hot, to reduce it to final thickness and to produce the final surface finish. The process is an interesting one for the control engineer, as it is multivariable (3 inputs, three outputs), strongly interacting, subject to constant disturbances (1 potentially measurable, 2 unmeasurable), and strongly constrained in both state and input variables. The process is shown diagramatically in Figure 1.

The strip enters the mill with thickness h_i at velocity v_i. In the first stand, the rolls, which are positioned by a screwdown drive (position s_1), exert a force F_1 on the strip, which deforms it plastically so that the strip exits with the thickness $h_{o,1}$ at velocity v_m. A second stand further reduces the strip to final thickness h_o and exit velocity v_o. The strip is rolled with a steady-state tensile stress t in the interstand section. This tension can be altered by a transient change in the peripheral speed of stand 2 from its steady-state value, (v_d), which is a third control input. The existence of rolling tension strongly couples the two stands. The input thickness h_i varies as a result of imperfections in the hot-rolling process which feeds it; a particular problem is caused by welds joining two hot-rolled strips to allow continuous cold processing, which cause step disturbances. This disturbance is potentially measurable. A major determinant of rolling force is the coefficient of friction between rolls and strip; this is never precisely known, varies with lubrication and roll surface condition, and is different between the two stands. The two coefficients (μ_1 and μ_2) are treated as constant disturbances which cannot be measured.

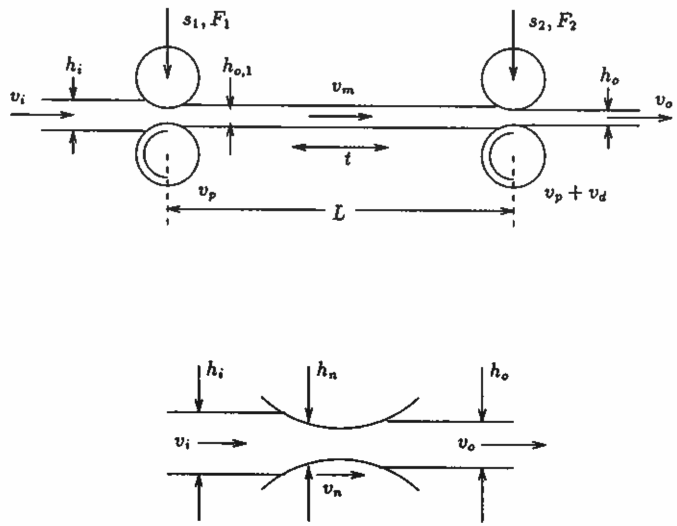

Figure 1: The rolling process

Modelling the Process

Roll forces

The rolling force, torque, and *neutral plane thickness* are calculated by integrating the forces and relative roll-strip velocities across the roll gap. The *neutral plane*, at which roll and strip have the same velocity, plays an important role in the calculation and knowledge of the thickness at this point is needed for dynamic modelling. The output variables, in our case F and h_n as we are assuming infinitely stiff roll drives and are not interested in torque, are given by

$$F = f(h_i, h_o, t_i, t_o, \mu)$$
$$h_n = h(h_i, h_o, t_i, t_o, \mu)$$

We will use a model linearized about the operating point, and will assume the mill input tension and the mill output tension are independently regulated and constant. Each variable is normalized to a deviation from its steady-state value; these variables will be indicated by hats, e.g. $\hat{F}$, $\hat{h}$ and so on.

Screw position and mill stretch

The output thickness from a stand is controlled by its screw position s, but the stand housings deflect under the very large forces used, so that roll force also influences output thickness;

$$h_o = \frac{F}{M} + s$$

Note that screw motion has been defined as positive upward. M is the modulus of elasticity of the housing (unit force/unit deflection).

Continuity of mass flow

Continuity demands that for each roll gap

$$v_i h_i = v_n h_n = v_o h_o$$

which, when linearized, provides additional relations which must be satisfied.

Dynamic relationships

Finally, we have two dynamic relationships which must be satisfied. The strip leaving the first stand enters the second stand T seconds later, where T is the travel time between stands. It follows that

$$h_{i,2}(t) = h_{o,1}(t - T)$$

In addition, if the two stands have differing velocities, the interstand strip will have a strain rate $(v_{i,2} - v_{o,1})/L$, and thus tensile stress will be given by

$$\dot{t} = \frac{E}{L}(v_{i,2} - v_{o,2})$$

We combine all these relationships, after linearizing about the operating point, to get a dynamic model. The model is a greatly simplified one, as we have assumed ideal drives (no dynamics), have taken them as infinitely stiff, and have assumed external tensions to be perfectly regulated. It is adequate, however, for preliminary investigations of mill control.

The Working Model

Model simplification and discretization

In using the above basic equations to formulate a working model, it is found that the tension time constant is quite short (15 ms) compared to the interstand time delay. Although the drives have been idealized, it is also impossible, with the forces available, to accelerate the large roll masses in such a short time. The tension time constant has therefore been ignored. The dominant dynamics are thus a 1-second time delay. The mill will be computer-controlled, and the model is accordingly discretized with a sampling interval of 0.1 seconds (determined by the characteristics of the X-ray thickness gauges used) and the drive velocity input commands are assumed constant over this interval.

Model matrices

The system is of the form

$$\begin{aligned} \mathbf{x}_{n+1} &= \mathbf{A}\mathbf{x}_n + \mathbf{B}\mathbf{u}_n + \mathbf{E}\mathbf{e}_n \\ \mathbf{y}_n &= \mathbf{C}\mathbf{x}_n + \mathbf{D}\mathbf{u}_n + \mathbf{F}\mathbf{e}_n \end{aligned}$$

where $\mathbf{x}$ is the 10×1 state vector, $\mathbf{u}$ is the 3×1 control vector, and $\mathbf{e}$ is the 3×1 disturbance vector.

The state vector

$$\mathbf{x}'(n) = (\hat{h}_{o,1}(n), \ldots, \hat{h}_{o,1}(n-9))$$

The control vector is made up of the three normalized velocity commands

$$\mathbf{u}'(n) = (s_1(n), s_2(n), v_d(n))$$

The output vector

$$\mathbf{y}'(n) = (\hat{h}_{o,1}(n), \hat{h}_o, \hat{t}(n), \hat{F}_1(n), \hat{F}_2)$$

The disturbance vector

$$\mathbf{e}'(n) = (\hat{h}_i(n), \hat{\mu}_1(n), \hat{\mu}_2(n))$$

All the variables except $\mathbf{u}$ are normalized. The variables s_1 and s_2 have units of 0.001 in: $\hat{v}_d$ is normalized. The saturation limits are actually on the velocities rather than the controls themselves, that is,

$$\dot{\mathbf{u}}'_{max} = (.018\text{in/sec}, .018\text{in/sec}, .07/\text{sec})$$

The numerical values for the matrices are given in Table 1.

Control Objectives

Outputs to be regulated

There are five possible outputs to be regulated; since only three are possible with three controls, we must make a choice. Two are obvious. The output thickness must be regulated, since this is the specified product. The tensile stress in the strip must also be constrained for

Table 1. SYSTEM MATRICES

MATRIX A

Columns 1 through 7

0	0	0	0	0	0	0
1.0000	0	0	0	0	0	0
0	1.0000	0	0	0	0	0
0	0	1.0000	0	0	0	0
0	0	0	1.0000	0	0	0
0	0	0	0	1.0000	0	0
0	0	0	0	0	1.0000	0
0	0	0	0	0	0	1.0000
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Columns 8 through 10

0	0	0.1120
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
1.0000	0	0
0	1.0000	0

MATRIX B

2.7600	-1.3500	-0.4600
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0

MATRIX C

Columns 1 through 7

1.0000	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Columns 8 through 10

0	0	0
0	0	0.8940
0	0	-16.9300
0	0	0.0700
0	0	0.3980

MATRIX D

0	0	0
-0.2230	1.8500	-0.5420
28.3000	204.0000	68.7000
-5.2100	-0.8430	-0.2850
-0.1010	-6.7500	-0.2460

MATRIX E

0.7130	0.5560	-0.1830
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0

MATRIX F

0	0	0
0.0025	-0.0250	0.2220
-0.3220	3.2200	27.6000
0.4440	0.3470	-0.0150
0.0020	-0.0120	-0.8600

safety reasons (the strip may not be allowed to go slack or break) and an easy way to constrain it is to regulate it. The best third choice is not so obvious. We could choose either the interstand thickness or some linear combination of the two forces.

Performance desired

We shall adapt as our test input a step disturbance $\hat{h}_i = 0.1$ occurring at $t = 0$. For this input, we wish to;

1. minimize the total length of the strip for which $|\hat{h}_o| > 0.04$
2. ensure that $|\hat{t}| < 1$ throughout the transient, and minimize its maximum value if possible subject to the output thickness specification
3. not saturate any drive at any point in the transient
4. minimize, but not necessarily regulate, the steady-state value of $\hat{F}_2$.

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Aero-Control

Problem #90-05 (Control of Missile Autopilot)
Problem #90-06 (Lower Order Controller Design for a High Order Plant)

Problem #90-05 CONTROL OF MISSILE AUTOPILOT

(A) General Description

It is desired to control the vertical acceleration of a rigid guided missile over different operating conditions. The missile is open loop stable, but has insufficient damping and is non-minimum phase.

(B) Reference

For the source of this problem contact U. Hartmann.

(C) Problem Description

The behaviour of a rigid body guided missile may be described as follows:

$$\dot{x} = Ax + bu$$
$$y = Cx$$
$$e = y - y_{ref}$$

where $x \in R^3$, $u \in R^1$, $y \in R^1$, $e \in R^1$ are given as follows:

u = elevator command (radians)
y = vertical acceleration (m/sec^2) (a_z in Figure 1)
x_1 = pitch rate (radians/sec) (Δq in Figure 1)
x_2 = angle of attack (radians) ($\Delta\alpha$ in Figure 1)
x_3 = elevator deflection angle (radians) ($\Delta\eta$ in Figure 1)
e = error between output and desired set point y_{ref}

It is desired to find a controller to control the vertical acceleration y subject to the following constraints:

(a) For a set point change in y_{ref}, the % overshoot in $y \leq 10\%$ and the steady-state error $\leq 5\%$. The bandwidth of the closed loop system ≤ 10 Hz.

(b) The elevator deflection angle x_3 should be limited to $\|x_3\| \leq 20°$. The elevator deflection angle rate $\dot{x}_3$ should be limited to $\|\dot{x}_3\| \leq 600°$/sec.

(c) The above control specifications are to be satisfied for 10 given flight operating conditions (which depend on mach number, altitude and mass of the vehicle).

In this vehicle, the pitch rate x_1 can be measured and used for purposes of control.

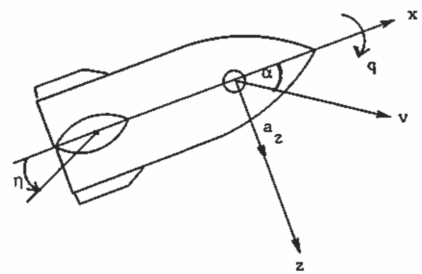

Figure 1: Notation used to describe vehicle.

The data for this problem is given as follows:

$$A = \begin{pmatrix} a_{11} & a_{12} & b_1 \\ 1 & a_{22} & b_2 \\ 0 & 0 & -190 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 0 \\ 190 \end{pmatrix}$$
$$c = \begin{pmatrix} c_{11} & c_{12} & d_1 \end{pmatrix}$$

where a_{11}, a_{12}, a_{22}, b_1, b_2, c_{11}, c_{12}, d_1 are given as follows for 10 different operating conditions (see Table 1).

Table 1

altitude	0 m					12000 m				
mach number	0.8		1.6		2.5	2.0		3.0		4.0
a_{11}	-0.327	0.391	-0.688	-0.738	-0.886	-1.364	-0.333	-0.337	-0.369	-0.402
a_{12}	-63.94	-130.29	-619.27	-651.57	-1068.85	-92.82	-163.24	-224.03	-253.71	-277.2
a_{22}	-1.0	-1.42	-2.27	-2.75	-3.38	-4.68	-0.666	-0.663	-0.80	-0.884
b_1	-155.96	-186.5	-552.9	-604.18	-1004.39	-128.46	-153.32	-228.72	-249.87	-419.35
b_2	-0.237	-0.337	-0.429	-0.532	-0.582	-0.087	-0.124	-0.112	-0.135	-0.166
c_{11}	0.326	0.35	0.65	0.66	0.79	1.36	0.298	0.319	0.33	0.36
c_{12}	-208.5	-272.38	-651.11	-913.64	-1926.45	-184.26	-247.75	-375.75	-500.59	-796.18
d_1	90.93	75.06	283.44	250.5	402.96	76.43	63.77	117.4	103.76	178.59

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Problem #90-06 LOWER ORDER CONTROLLER DESIGN FOR A HIGH ORDER PLANT

(A) Motivation for the Problem

The motivation for the problem is to provide a uniform testbed for the evaluation of different approaches and algorithms in design of low-order linear feedback controllers for a high-order plant (typical of most industrial problems). In this case, a design example from the aircraft industry is provided. The open-loop plant model possesses key features that render the application of conventional design techniques difficult, if not impossible, in achieving the specified design requirements. They are:

(i) High dynamic order (55^{th}-order);

(ii) Multivariable system with two inputs and two outputs;

(iii) Open-loop instability (i.e., Flutter mode);

(iv) Numerous highly oscillatory modes (i.e., lightly damped structural modes);

(v) Nonminimum-phase characteristics.

(B) Brief Description of Problem Origin

This plant is an aeroelastic model for a modified Boeing B-767 airplane, at a flutter condition, that was used in a research study of active control technology [1] where a linear-quadratic-gaussian (LQG)-based design has been developed for flutter control and gust load alleviation. The resulting 55^{th}-order controller has been subsequently reduced to a 10^{th}-order controller for practical implementation using a standard modal residualization technique. Description of the design philosophy and related details on final design performance can be found in reference [1] and [2]. Additional results on controller reduction applied to this problem can be found in reference [3].

(C) Problem Description

The problem is to synthesize a linear feedback control-law of low order (preferably less than 10) that achieves the following design objectives:

(i) Stabilizes the flutter mode with damping of at least 0.015;

(ii) Provides minimal damping of 0.4 to the remaining low-frequency modes;

(iii) Reduces the mean-square responses of aircraft dynamic load variables to turbulence;

(iv) Possesses desirable robustness in terms of phase and gain margins at each control loop.

The following are a set of design requirements for the benchmark problem:

(i) Minimum damping of 0.015 for the flutter mode identified by its frequency near $\omega \approx 20$ rad/sec (e.g., flutter mode damping achieved by the LQG controller in references [1] and [2] is 0.074);

(ii) Minimum damping of 0.40 for the remaining low-frequency modes (e.g., aircraft short-period mode, compensator modes, etc.);

(iii) Adequate attenuation of aircraft dynamic load responses to a 10ft/sec rms turbulence w_1 (e.g., comparable with those achieved by the LQG controller in references [1] and [2]);

(iv) Moderate control activities based on mean-square responses of control deflections and their rates to a 10ft/sec turbulence $w_1(t)$ (i.e., comparable with results in references [1] and [2]);

(v) Adequate phase and gain margins:

- Gain margin of ± 6 dB
- Phase margin of ±45°

in the elevator and aileron control lops. Stability margins are evaluated one-loop-at-a-time using classical single-loop analysis).

The linear feedback control system could be designed *directly*, or from the *reduction of a high-order controller* (for example, using a previously designed LQG controller of references [1] and [2] meets the above design objectives). The LQG controller design in references [1] and [2] was obtained from the following quadratic cost function with process and sensor noise characteristics for the plant model described in Appendix I.

- **Design cost function:** The cost function is of the form:

$$J = \int_0^\infty [Q_1 y_1^2(t) + Q_2 y_2^2(t) + Q_3 y_3^2(t) + R_1 u_1^2(t) + R_2 u_2^2(t)]dt$$

The following output variables y_i $(i = 1, 2, 3)$ have been penalized in the cost function with weighting factors Q_i $(i = 1, 2, 3)$:

Output Variables y_i	Weighting Factor Q_i
(1) Inboard bending moment (BMOMI)	3.76×10^{-14}
(2) Inboard torsion (TORI)	1.20×10^{-13}
(3) Outboard torsion (TORO)	2.45×10^{-12}

along with the input variables u_i and their weighting factors R_i $(i = 1, 2)$:

Input Variable u_i	Weighting Factor R_i
(1) Elevator control (ELEVC)	3.647×10^2
(2) Aileron control (AILC)	1.459×10^1

- **Process and sensor noise characteristics:** A Kalman filter was designed using the measurement of aircraft pitch rate QCG and wing-tip acceleration WTIPDD. The process noises w_i ($i = 1,2,3$) have input distribution matrix B_n and spectral densities W_i ($i = 1,2,3$):

Process Noise w_i	Spectral Density W_i
(1) Vertical gust (WG)	2.8224×10^4 (in/sec)^2/rad/sec)
(2) Elevator input noise (ELEVN)	2.742×10^{-5}(rad)^2/(rad/sec)
(3) Aileron input noise (AILN)	6.854×10^{-4}(rad)^2/(rad/sec)

The sensor noises v_i ($i = 1,2$) for the above two measurements have the following spectral densities:

Sensor Noise v_i	Spectral Density V_i
(1) Pitch rate (QCG)	6.85×10^{-6}(rad)^2/(rad/sec)
(2) Wing-tip acceleration (WTIPDD)	3.73×10^2 (in/sec^2)^2/(rad/sec)

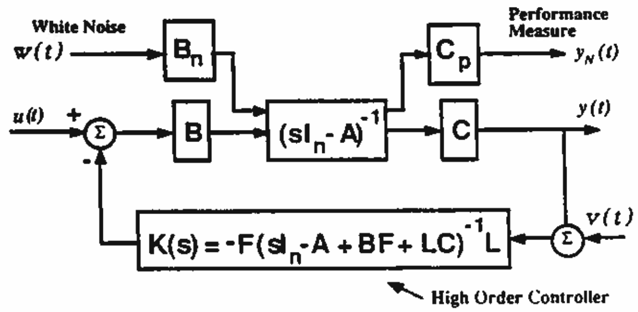

Fig.1 Closed-loop system structure with a full order controller

Appendix I shows the resulting gain matrices F and L of the regulator and Kalman filter design respectively, using the above design parameters. Figure 1 shows a block diagram of the closed-loop system constructed with a full-order LQG controller where the matrices A, B, C, B_n, C_p, F and L are given in Appendix I.

For convenience, design evaluation of the above full-order LQG controller, a 10^{th}-order (reduced) controller [1,2] and a 5^{th}-order controller [3] are given in Table 1.

Table 1

Closed-Loop Evaluation for Full (LQG Design) and Reduced Order Controllers

Controller Order	55^{th}[2]	10^{th}[2]	5^{th}[3,LCFW]
Flutter Mode Damping	0.074	0.070	0.0501
Inboard Wing Station:†			
Bending Moment ($\times 10^5$ in-lbs)	2.348	2.349	2.624
Shear (lbs)	854.	861.	950.
Torsion ($\times 10^4$ in-lbs)	4.437	4.597	4.969
Outboard Wing Station:†			
Bending Moment ($\times 10^5$ in-lbs)	0.259	0.262	0.239
Shear (lbs)	259.	260.	286.
Torsion ($\times 10^4$ in-lbs)	1.196	1.206	1.274
Control Surface Activities:†			
Elevator Control Deflection (rad)	0.00054	0.00056	0.000339
Elevator Control Surface Rate (rad/sec)	0.00407	0.00436	0.00401
Aileron Control Deflection (rad)	0.00158	0.00163	0.00176
Aileron Control Surface Rate (rad/sec)	0.0275	0.0276	0.0332
Stability Margins:*			
Elevator Control Loop: Gain Margin (dB)	15.9	36.4	18.2
Phase Margin (deg)	180	180	-102
Aileron Control Loop: Gain Margin (dB)	14	25	(-15,14.4)
Phase Margin (deg)	58.6	(-138.9,146)	(-140,82.3)

† Root-Mean-Square (rms) to a 10ft/sec vertical Dryden turbulence [by injecting white-noise at the input $w_1(t)$ with a power spectral density of (10ft/sec)^2(rad/sec)].

* Frequency range considered is $0.01 \le w \le 100$ rad/sec.

(D) Previous Results Obtained

Discussion of the aircraft aeroelastic model and design in the LQG Controller can be found in the following references:

1. Ly, U.-L. and D. Gangsaas, "Application of Modified Linear Quadratic Guassian Design to Active Control of a Transport Airplane", *AIAA*, Guidance and Control Conference, Boulder, Colorado, August 1979.

2. Ly, U.-L., D. Gangsaas and D.C. Norman, "Practical Gust Load Aleviation and Flutter Suppression Control Laws Based on an LQG Methodology", *AIAA*, Aerospace Sciences Meeting, 1981.

A discussion of some controller reduction methods used on the full order LQG designed controller to generate lower order controllers appears in the following:

3. Liu, Y., B.D.O. Anderson, U.-L. Ly, "Coprime Factorization Controller Reduction with Bezout Identity Induced Frequency Weighting", *Automatica*, to appear.

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Appendix I
Plant Model State Matrices

Aeroelastic model of a modified B767-aircraft at a flutter condition is a linear state model of the form

$$\dot{x}(t) = Ax(t) + Bu(t) + B_n w(t)$$

where

- $x(t)$ is a state vector (of dimension 55) containing the aircraft rigid and aeroelastic modes, actuator modes, Dryden turbulence filter and other noise shaping filters;
- $u(t)$ is a control vector (of dimension 2) containing the elevator control ELEVC and the aileron control AILC;
- $w(t)$ is a process noise input vector (of dimension 3) containing the vertical turbulence input $w_1(t)$, the elevator noise input $w_2(t)$ and the aileron noise input $w_3(t)$.

The sensor output $y(t) = [\text{QCG} \quad \text{WTIPDD}]^T$ consists of the aircraft pitch rate QCG and the wing-tip acceleration WTIPDD variables and is given by

$$y(t) = Cx(t) + v(t)$$

The performance output variables

$$y_N(t) = [\text{SHRI,BMOMI,TORI,SHRO,BMOMO,TORO,ELD,EL,AILOD,AILO}]^T$$

given by

$$y_N(t) = C_p x(t)$$

represent those outputs to be used in the covariance analysis of the aircraft subjected to a vertical turbulence of 10ft/sec rms. They represent the dynamic loads of the aircraft at the inboard and outboard wing stations, the elevator and aileron control deflections and rates respectively, i.e.,

SHRI:	Shear force at the inboard wing station (lbs)
BMOMI:	Bending moment at the inboard wing station (in-lbs)
TORI:	Torsion at the inboard wing station (in-lbs)
SHRO:	Shear force at the outboard wing station (lbs)
BMOMO:	Bending moment at the outboard wing station (lbs)
TORO:	Torsion at'the outboard wing station (in-lbs)
ELD:	Elevator control surface rate (rad/sec)
EL:	Elevator control surface deflection (rad)
AILOD:	Aileron control surface rate (rad/sec)
AILO:	Aileron control surface deflection (rad)

A closed-loop system with a full-order LQG controller is shown in Figure 1. State matrices A, B, B_n, C and C_p along with the full-state feedback gain matrix F and the Kalman gain matrix L corresponding to the LQG controller in [1,2] are shown below.

A

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
X1	0.1015	-19.77	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X2	19.77	0.1015	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X3	0.0000E+00	0.0000E+00	-2.3202E-02	-9.2543E-02	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X4	0.0000E+00	0.0000E+00	9.2543E-02	-2.3202E-02	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X5	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-0.3165	-14.33	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X6	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	14.33	-0.3165	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X7	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-0.3892	-22.29	0.0000E+00	0.0000E+00
X8	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	22.29	-0.3892	0.0000E+00	0.0000E+00
X9	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-0.9883	-36.16
X10	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	36.16	-0.9883
X11	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X12	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X13	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X14	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X15	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X16	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X17	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X18	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X19	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X20	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X21	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X22	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X23	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X24	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X25	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X26	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X27	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X28	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X29	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X30	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X31	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X32	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X33	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X34	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X35	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X36	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X37	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X38	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X39	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X40	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X41	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X42	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X43	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X44	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X45	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ELEV	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ELEVD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ELEVDD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AIL	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILDD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
WIND1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
WIND2	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ENOISE	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ANOISE	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

	X11	X12	X13	X14	X15	X16	X17	X18	X19	X20
X1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X2	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X3	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X4	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X5	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X6	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X7	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X8	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X9	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X10	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X11	-1.342	-2.547	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X12	2.547	-1.342	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X13	0.0000E+00	0.0000E+00	-2.312	-21.51	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X14	0.0000E+00	0.0000E+00	21.51	-2.312	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X15	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-2.792	-26.71	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X16	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	26.71	-2.792	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X17	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-2.796	-63.79	0.0000E+00	0.0000E+00
X18	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	63.79	-2.796	0.0000E+00	0.0000E+00
X19	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-3.332	-56.37
X20	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	56.37	-3.332

X21	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+C
X22	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+C
X23	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+C
X24	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+C
X25	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+C
X26	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+C
X27	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+C
X28	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X29	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X30	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X31	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X32	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X33	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X34	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X35	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X36	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X37	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X38	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X39	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X40	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X41	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X42	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X43	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X44	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X45	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
ELEV	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
ELEVD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
ELEVDD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
AIL	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
AILD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
AILDD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
WIND1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
WIND2	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
ENOISE	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
ANOISE	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0

	X21	X22	X23	X24	X25	X26	X27	X28	X29	X30
X1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X2	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X3	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X4	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+0
X5	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X6	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X7	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X8	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X9	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X10	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X11	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X12	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X13	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X14	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X15	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X16	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X17	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X18	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X19	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X20	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X21	-3.418	-88.67	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X22	88.67	-3.418	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X23	0.0000E+00	0.0000E+00	-4.441	-69.45	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X24	0.0000E+00	0.0000E+00	69.45	-4.441	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X25	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-5.108	-53.39	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X26	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	53.39	-5.108	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X27	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-5.199	-50.64	0.0000E+00	0.0000E+00
X28	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	50.64	-5.199	0.0000E+00	0.0000E+00
X29	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-5.301	0.0000E+00
X30	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-5.670
X31	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	92.96
X32	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X33	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X34	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X35	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X36	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X37	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X38	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X39	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X40	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X41	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X42	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X43	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X44	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X45	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ELEV	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

ELEVD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ELEVDD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AIL	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILDD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
WIND1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
WIND2	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ENOISE	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ANOISE	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

	X31	X32	X33	X34	X35	X36	X37	X38	X39	X40
X1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X2	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X3	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X4	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X5	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X6	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X7	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X8	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X9	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X10	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X11	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X12	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X13	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X14	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X15	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X16	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X17	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X18	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X19	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X20	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X21	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X22	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X23	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X24	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X25	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X26	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X27	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X28	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X29	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X30	-92.96	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X31	-5.670	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X32	0.0000E+00	-6.197	-40.00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X33	0.0000E+00	40.00	-6.197	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X34	0.0000E+00	0.0000E+00	0.0000E+00	-8.177	-16.61	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X35	0.0000E+00	0.0000E+00	0.0000E+00	16.61	-8.177	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X36	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-8.221	-139.1	0.0000E+00	0.0000E+00	0.0000E+00
X37	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	139.1	-8.221	0.0000E+00	0.0000E+00	0.0000E+00
X38	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-10.35	-163.0	0.0000E+00
X39	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	163.0	-10.35	0.0000E+00
X40	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-11.72
X41	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	109.3
X42	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X43	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X44	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X45	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ELEV	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ELEVD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ELEVDD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AIL	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILDD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
WIND1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
WIND2	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ENOISE	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ANOISE	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

	X41	X42	X43	X44	X45	ELEV	ELEVD	ELEVDD	AIL	AILD
X1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-1970.	6.796	-0.1334	-1429.	18.03
X2	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-2665.	7.196	-0.1568	-2222.	-2.684
X3	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	586.9	1.7334E-03	-4.3119E-05	119.0	4.8611E-04
X4	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-501.1	4.1488E-03	-8.0848E-05	-105.9	1.0274E-03
X5	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	134.5	-0.2690	4.9694E-03	-146.8	-0.7097
X6	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-164.5	0.4627	-8.4320E-03	196.0	1.345
X7	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	608.5	-1.835	3.6978E-02	175.1	-1.082
X8	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-77.54	0.1595	-2.0525E-03	9.616	0.6370
X9	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-1117.	4.615	-8.4286E-02	-34.14	6.425
X10	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	1790.	-3.752	7.1871E-02	-222.7	1.899
X11	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-1.3858E+04	0.5550	-1.2727E-02	-2014.	-5.0439E-02
X12	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	1.5010E+04	-3.138	6.5016E-02	2668.	-1.060
X13	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-2.1697E+04	61.46	-1.286	-3652.	5.074
X14	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	1.9966E+04	-53.56	1.121	2881.	-26.77

X15	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-2861.	13.14	-0.2426	-1080.	48.49
X16	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	6583.	-19.32	0.3841	-3883.	-12.93
X17	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-2212.	-12.28	0.1229	-1647.	-5.130
X18	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	6321.	-27.17	0.2049	-1045.	-15.59
X19	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	1.3964E+04	-17.24	3.5851E-02	562.0	-25.48
X20	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	2382.	-20.08	0.3605	902.6	-1.623
X21	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-303.7	2.624	-0.1208	1129.	4.399
X22	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	542.8	0.5789	0.1194	-1258.	-6.993
X23	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-527.1	-36.01	0.3072	-1267.	7.583
X24	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	2658.	3.812	-3.7101E-02	-49.11	-4.347
X25	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	2.5341E+04	-36.45	0.4514	4360.	-131.3
X26	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	6538.	-28.56	0.3244	3790.	90.71
X27	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	4.2048E+04	-59.43	0.7558	2501.	-17.39
X28	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	210.6	-39.51	0.5076	-470.4	-18.75
X29	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X30	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	994.2	13.57	-2.1971E-02	828.4	12.90
X31	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	769.2	10.60	-9.2043E-02	810.0	19.07
X32	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	4.2702E+04	-120.7	1.940	-3077.	35.90
X33	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-7.6982E+04	140.6	-2.796	-348.3	-38.56
X34	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-288.8	-0.9428	2.3831E-02	-1499.	0.9758
X35	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-2237.	8.455	-0.1565	-6578.	49.50
X36	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-1.6942E+04	94.43	-1.144	-60.89	0.2221
X37	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-1.2513E+04	68.02	-0.6821	-29.37	0.4639
X38	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	5748.	-35.78	0.2067	-39.31	-0.6830
X39	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-1.4657E+04	216.8	-0.6615	123.4	9.043
X40	-109.3	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-4089.	-258.1	-1.059	33.03	-2.040
X41	-11.72	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	7940.	309.0	1.175	61.04	2.311
X42	0.0000E+00	-11.79	-304.6	0.0000E+00	0.0000E+00	9582.	37.30	-3.1126E-02	140.9	2.627
X43	0.0000E+00	304.6	-11.79	0.0000E+00	0.0000E+00	9474.	41.40	-5.0827E-02	156.3	3.098
X44	0.0000E+00	0.0000E+00	0.0000E+00	-33.27	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
X45	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-221.2	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ELEV	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	1.000	0.0000E+00	0.0000E+00	0.0000E+00
ELEVD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	1.000	0.0000E+00	0.0000E+00
ELEVDD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-8.0000E+05	-6.0800E+04	-1060.	0.0000E+00	0.0000E+00
AIL	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	1.000
AILD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILDD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-8.0000E+05	-6.0800E+04
WIND1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
WIND2	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ENOISE	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ANOISE	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

	AILDD	WIND1	WIND2	ENOISE	ANOISE
X1	-0.1517	0.0000E+00	-3.158	0.0000E+00	0.0000E+00
X2	-5.0292E-02	0.0000E+00	1.779	0.0000E+00	0.0000E+00
X3	-7.4279E-06	0.0000E+00	5.3132E-03	0.0000E+00	0.0000E+00
X4	-1.1816E-05	0.0000E+00	8.0111E-03	0.0000E+00	0.0000E+00
X5.	1.2317E-03	0.0000E+00	0.1066	0.0000E+00	0.0000E+00
X6	-3.9645E-03	0.0000E+00	-0.4078	0.0000E+00	0.0000E+00
X7	1.1391E-02	0.0000E+00	0.1001	0.0000E+00	0.0000E+00
X8	-2.8823E-03	0.0000E+00	-0.1571	0.0000E+00	0.0000E+00
X9	-4.9071E-02	0.0000E+00	-1.060	0.0000E+00	0.0000E+00
X10	-1.5192E-02	0.0000E+00	0.4854	0.0000E+00	0.0000E+00
X11	1.0263E-03	0.0000E+00	-2.733	0.0000E+00	0.0000E+00
X12	1.1647E-02	0.0000E+00	2.067	0.0000E+00	0.0000E+00
X13	-0.1277	0.0000E+00	3.001	0.0000E+00	0.0000E+00
X14	0.2540	0.0000E+00	3.544	0.0000E+00	0.0000E+00
X15	-0.3712	0.0000E+00	-7.199	0.0000E+00	0.0000E+00
X16	1.0825E-02	0.0000E+00	7.207	0.0000E+00	0.0000E+00
X17	2.0187E-02	0.0000E+00	0.6186	0.0000E+00	0.0000E+00
X18	6.4588E-02	0.0000E+00	1.076	0.0000E+00	0.0000E+00
X19	0.1072	0.0000E+00	2.821	0.0000E+00	0.0000E+00
X20	7.7575E-02	0.0000E+00	-0.7866	0.0000E+00	0.0000E+00
X21	-2.7240E-02	0.0000E+00	-0.1111	0.0000E+00	0.0000E+00
X22	2.9297E-02	0.0000E+00	1.509	0.0000E+00	0.0000E+00
X23	-6.5626E-03	0.0000E+00	-0.3659	0.0000E+00	0.0000E+00
X24	-8.4969E-03	0.0000E+00	0.8635	0.0000E+00	0.0000E+00
X25	0.8394	0.0000E+00	7.433	0.0000E+00	0.0000E+00
X26	-0.3495	0.0000E+00	-15.36	0.0000E+00	0.0000E+00
X27	0.1113	0.0000E+00	4.570	0.0000E+00	0.0000E+00
X28	0.1395	0.0000E+00	-0.2771	0.0000E+00	0.0000E+00
X29	0.0000E+00	0.0000E+00	-1.251	0.0000E+00	0.0000E+00
X30	-3.6967E-02	0.0000E+00	-0.5031	0.0000E+00	0.0000E+00
X31	-8.1012E-02	0.0000E+00	-0.4524	0.0000E+00	0.0000E+00
X32	-0.2467	0.0000E+00	-1.592	0.0000E+00	0.0000E+00
X33	0.1981	0.0000E+00	1.956	0.0000E+00	0.0000E+00
X34	-4.1943E-02	0.0000E+00	-5.981	0.0000E+00	0.0000E+00
X35	-0.4980	0.0000E+00	-10.88	0.0000E+00	0.0000E+00
X36	-1.1955E-02	0.0000E+00	-1.121	0.0000E+00	0.0000E+00
X37	-9.9611E-03	0.0000E+00	0.1532	0.0000E+00	0.0000E+00
X38	7.8189E-03	0.0000E+00	-0.4254	0.0000E+00	0.0000E+00
X39	-8.7434E-02	0.0000E+00	-0.7886	0.0000E+00	0.0000E+00
X40	2.3536E-02	0.0000E+00	-0.1053	0.0000E+00	0.0000E+00

X41	-2.8729E-02	0.0000E+00	-2.007	0.0000E+00	0.0000E+00
X42	-1.8542E-02	0.0000E+00	0.1886	0.0000E+00	0.0000E+00
X43	-2.2171E-02	0.0000E+00	9.2049E-03	0.0000E+00	0.0000E+00
X44	0.0000E+00	0.0000E+00	-17.07	0.0000E+00	0.0000E+00
X45	0.0000E+00	0.0000E+00	-37.83	0.0000E+00	0.0000E+00
ELEV	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ELEVD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ELEVDD	0.0000E+00	0.0000E+00	0.0000E+00	-1.6000E+07	0.0000E+00
AIL	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILD	1.000	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILDD	-1060.	0.0000E+00	0.0000E+00	0.0000E+00	-1.6000E+07
WIND1	0.0000E+00	0.0000E+00	-0.2668	0.0000E+00	0.0000E+00
WIND2	0.0000E+00	1.000	-1.033	0.0000E+00	0.0000E+00
ENOISE	0.0000E+00	0.0000E+00	0.0000E+00	-20.00	0.0000E+00
ANOISE	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-20.00

B

	ELEVC	AILC
X1	0.0000E+00	0.0000E+00
X2	0.0000E+00	0.0000E+00
X3	0.0000E+00	0.0000E+00
X4	0.0000E+00	0.0000E+00
X5	0.0000E+00	0.0000E+00
X6	0.0000E+00	0.0000E+00
X7	0.0000E+00	0.0000E+00
X8	0.0000E+00	0.0000E+00
X9	0.0000E+00	0.0000E+00
X10	0.0000E+00	0.0000E+00
X11	0.0000E+00	0.0000E+00
X12	0.0000E+00	0.0000E+00
X13	0.0000E+00	0.0000E+00
X14	0.0000E+00	0.0000E+00
X15	0.0000E+00	0.0000E+00
X16	0.0000E+00	0.0000E+00
X17	0.0000E+00	0.0000E+00
X18	0.0000E+00	0.0000E+00
X19	0.0000E+00	0.0000E+00
X20	0.0000E+00	0.0000E+00
X21	0.0000E+00	0.0000E+00
X22	0.0000E+00	0.0000E+00
X23	0.0000E+00	0.0000E+00
X24	0.0000E+00	0.0000E+00
X25	0.0000E+00	0.0000E+00
X26	0.0000E+00	0.0000E+00
X27	0.0000E+00	0.0000E+00
X28	0.0000E+00	0.0000E+00
X29	0.0000E+00	0.0000E+00
X30	0.0000E+00	0.0000E+00
X31	0.0000E+00	0.0000E+00
X32	0.0000E+00	0.0000E+00
X33	0.0000E+00	0.0000E+00
X34	0.0000E+00	0.0000E+00
X35	0.0000E+00	0.0000E+00
X36	0.0000E+00	0.0000E+00
X37	0.0000E+00	0.0000E+00
X38	0.0000E+00	0.0000E+00
X39	0.0000E+00	0.0000E+00
X40	0.0000E+00	0.0000E+00
X41	0.0000E+00	0.0000E+00
X42	0.0000E+00	0.0000E+00
X43	0.0000E+00	0.0000E+00
X44	0.0000E+00	0.0000E+00
X45	0.0000E+00	0.0000E+00
ELEV	0.0000E+00	0.0000E+00
ELEVD	0.0000E+00	0.0000E+00
ELEVDD	8.0000E+05	0.0000E+00
AIL	0.0000E+00	0.0000E+00
AILD	0.0000E+00	0.0000E+00
AILDD	0.0000E+00	8.0000E+05
WIND1	0.0000E+00	0.0000E+00
WIND2	0.0000E+00	0.0000E+00
ENOISE	0.0000E+00	0.0000E+00
ANOISE	0.0000E+00	0.0000E+00

B_n

	w_1	w_2	w_3
X1	0.0000E+00	0.0000E+00	0.0000E+00
X2	0.0000E+00	0.0000E+00	0.0000E+00
X3	0.0000E+00	0.0000E+00	0.0000E+00
X4	0.0000E+00	0.0000E+00	0.0000E+00
X5	0.0000E+00	0.0000E+00	0.0000E+00
X6	0.0000E+00	0.0000E+00	0.0000E+00
X7	0.0000E+00	0.0000E+00	0.0000E+00
X8	0.0000E+00	0.0000E+00	0.0000E+00
X9	0.0000E+00	0.0000E+00	0.0000E+00
X10	0.0000E+00	0.0000E+00	0.0000E+00
X11	0.0000E+00	0.0000E+00	0.0000E+00
X12	0.0000E+00	0.0000E+00	0.0000E+00
X13	0.0000E+00	0.0000E+00	0.0000E+00
X14	0.0000E+00	0.0000E+00	0.0000E+00
X15	0.0000E+00	0.0000E+00	0.0000E+00
X16	0.0000E+00	0.0000E+00	0.0000E+00
X17	0.0000E+00	0.0000E+00	0.0000E+00
X18	0.0000E+00	0.0000E+00	0.0000E+00
X19	0.0000E+00	0.0000E+00	0.0000E+00
X20	0.0000E+00	0.0000E+00	0.0000E+00
X21	0.0000E+00	0.0000E+00	0.0000E+00
X22	0.0000E+00	0.0000E+00	0.0000E+00
X23	0.0000E+00	0.0000E+00	0.0000E+00
X24	0.0000E+00	0.0000E+00	0.0000E+00
X25	0.0000E+00	0.0000E+00	0.0000E+00
X26	0.0000E+00	0.0000E+00	0.0000E+00
X27	0.0000E+00	0.0000E+00	0.0000E+00
X28	0.0000E+00	0.0000E+00	0.0000E+00
X29	0.0000E+00	0.0000E+00	0.0000E+00
X30	0.0000E+00	0.0000E+00	0.0000E+00
X31	0.0000E+00	0.0000E+00	0.0000E+00
X32	0.0000E+00	0.0000E+00	0.0000E+00
X33	0.0000E+00	0.0000E+00	0.0000E+00
X34	0.0000E+00	0.0000E+00	0.0000E+00
X35	0.0000E+00	0.0000E+00	0.0000E+00
X36	0.0000E+00	0.0000E+00	0.0000E+00
X37	0.0000E+00	0.0000E+00	0.0000E+00
X38	0.0000E+00	0.0000E+00	0.0000E+00
X39	0.0000E+00	0.0000E+00	0.0000E+00
X40	0.0000E+00	0.0000E+00	0.0000E+00
X41	0.0000E+00	0.0000E+00	0.0000E+00
X42	0.0000E+00	0.0000E+00	0.0000E+00
X43	0.0000E+00	0.0000E+00	0.0000E+00
X44	0.0000E+00	0.0000E+00	0.0000E+00
X45	0.0000E+00	0.0000E+00	0.0000E+00
ELEV	0.0000E+00	0.0000E+00	0.0000E+00
ELEVD	0.0000E+00	0.0000E+00	0.0000E+00
ELEVDD	0.0000E+00	8.0000E+05	0.0000E+00
AIL	0.0000E+00	0.0000E+00	0.0000E+00
AILD	0.0000E+00	0.0000E+00	0.0000E+00
AILDD	0.0000E+00	0.0000E+00	8.0000E+05
WIND1	0.3713	0.0000E+00	0.0000E+00
WIND2	1.2450	0.0000E+00	0.0000E+00
ENOISE	0.0000E+00	1.0000	0.0000E+00
ANOISE	0.0000E+00	0.0000E+00	1.0000

C

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
QCG	4.4247E-05	4.3403E-05	4.9713E-05	4.5556E-05	-4.1311E-06	7.9201E-06	7.2496E-06	-6.7385E-05	-1.8236E-04	2.0657E-05
WTIPDD	35.92	-12.46	-0.5332	-0.5659	-0.9664	12.75	-21.45	6.822	33.88	17.00

	X11	X12	X13	X14	X15	X16	X17	X18	X19	X20
QCG	1.3194E-04	-1.5849E-04	-6.7655E-05	-8.7358E-06	-9.8119E-05	3.1119E-05	-5.7479E-05	3.1600E-04	-5.7750E-04	7.1000E-04
WTIPDD	-0.1380	-0.8803	5.465	-1.614	55.18	30.10	-4.878	14.36	62.54	55.18

	X21	X22	X23	X24	X25	X26	X27	X28	X29	X30
QCG	-9.6843E-05	2.9934E-05	5.4014E-04	-4.7620E-04	-4.1630E-06	-2.1609E-04	-1.0541E-04	-3.1883E-05	-1.5062E-04	-2.7714E-04
WTIPDD	-138.8	-104.4	6.831	-11.78	-55.04	-30.66	1.402	-30.33	4.869	-74.28

	X31	X32	X33	X34	X35	X36	X37	X38	X39	X40
QCG	-2.4361E-04	2.7990E-05	-1.6592E-04	1.2748E-05	-2.0762E-05	-1.2546E-04	-4.4360E-05	-1.0147E-05	1.5343E-04	2.1956E-05
WTIPDD	74.14	16.30	3.118	8.728	-74.53	1.460	-3.334	-7.367	-1.032	2.867

	X41	X42	X43	X44	X45	ELEV	ELEVD	ELEVDD	AIL	AILD
QCG	-2.9386E-05	6.5513E-05	6.1813E-05	1.9995E-05	8.9674E-06	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
WTIPDD	6.583	-1.361	1.350	10.71	-25.91	48.52	-7.932	7.9915E-02	-2.5618E+04	-167.4

	AILDD	WIND1	WIND2	ENOISE	ANOISE
QCG	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
WTIPDD	0.2213	0.0000E+00	1.880	0.0000E+00	0.0000E+00

C_p

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
SHRI	-280.4	-189.2	54.94	64.24	151.2	93.86	159.9	70.45	-118.0	-29.86
BMOMI	-1.0116E+04	-9214.	1.9756E+04	2.1545E+04	1.3993E+04	1.7694E+04	-2984.	-3842.	-1.7279E+04	1314.
TORI	-3.5918E+04	-1.8298E+04	1444.	2675.	1.6733E+04	7989.	3.4725E+04	1.1489E+04	3528.	9170.
SHRO	-1.008	-5.628	22.26	24.73	5.633	17.59	6.685	-1.131	-17.83	-4.323
BMOMO	8616.	-576.7	1146.	1214.	-870.0	3613.	-3028.	1084.	8362.	5560.
TORO	-3685.	1359.	546.4	605.9	71.67	-1539.	2550.	-599.8	2060.	1803.
ELD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
EL	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILOD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILO	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

	X11	X12	X13	X14	X15	X16	X17	X18	X19	X20
SHRI	102.8	101.8	6.202	-19.80	-204.1	-168.5	38.07	17.06	237.5	253.3
BMOMI	3.2923E+04	3.3817E+04	-1082.	3076.	-3.8871E+04	-3.6494E+04	-7038.	1.1182E+04	8002.	1.0350E+04
TORI	3849.	3825.	-3369.	-1.0423E+04	-4762.	-1033.	-0.8833	4619.	3448.	1303.
SHRO	37.01	38.72	-4.403	-3.027	-27.42	-23.76	-13.45	2.659	-49.80	-48.30
BMOMO	1935.	2152.	830.2	-882.3	1.1555E+04	9656.	660.9	2046.	8408.	7455.
TORO	786.8	911.5	-899.7	-540.4	-1427.	-1640.	-693.1	3405.	1955.	530.8
ELD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
EL	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILOD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILO	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

	X21	X22	X23	X24	X25	X26	X27	X28	X29	X30
SHRI	7.799	97.62	-29.15	5.759	-33.32	-7.012	-103.5	-139.6	-159.0	-326.0
BMOMI	-5.7674E+04	-5.8520E+04	-9678.	-1.7572E+04	-4136.	-493.0	152.7	-2391.	-5.4007E+04	1.4065E+04
TORI	-1.6288E+04	917.4	-6520.	-4420.	1.3953E+04	1.0935E+04	-1.2971E+04	2265.	-6697.	-6.6463E+04
SHRO	-37.52	-36.55	3.951	-4.938	6.345	5.479	25.88	34.74	-62.23	-10.69
BMOMO	-6012.	-5724.	-3113.	-2967.	1288.	583.2	-6625.	-5544.	-2987.	2461.
TORO	-1239.	-1711.	-5337.	-5925.	7784.	6242.	-8377.	-2130.	-2052.	8424.
ELD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
EL	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILOD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILO	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

	X31	X32	X33	X34	X35	X36	X37	X38	X39	X40
SHRI	287.0	14.71	-11.49	93.88	203.2	-1.189	1.220	1.539	0.8601	3.222
BMOMI	-1.8204E+04	-4219.	-1617.	4.2055E+04	3.3610E+04	95.83	-530.3	-423.9	-80.47	580.5
TORI	5.9803E+04	-4749.	-3525.	-6148.	2.4890E+04	137.6	-1595.	-2912.	-203.6	2433.
SHRO	10.05	-16.66	-2.834	55.84	28.73	0.2784	-1.598	-2.559	-0.3003	1.781
BMOMO	-3217.	2832.	324.2	7562.	-1.2172E+04	9.422	-27.31	0.5400	-0.7739	14.79
TORO	-8033.	159.6	-1013.	-4374.	9713.	-41.64	-7.796	20.11	25.14	56.85
ELD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
EL	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILOD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILO	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

	X41	X42	X43	X44	X45	ELEV	ELEVD	ELEVDD	AIL	AILD
SHRI	6.416	0.6052	-0.5647	-97.82	-1.384	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
BMOMI	667.4	-20.59	28.40	-3.2517E+04	-1345.	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
TORI	3134.	-170.6	202.4	-1.4519E+04	-691.2	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
SHRO	2.065	-0.1941	0.2183	-35.76	-1.258	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
BMOMO	5.506	-2.632	2.610	1184.	-38.55	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
TORO	-202.9	1.613	1.493	-7454.	-471.2	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ELD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	1.000	0.0000E+00	0.0000E+00	0.0000E+00
EL	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	1.000	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILOD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	1.000
AILO	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	1.000	0.0000E+00

	AILDD	WIND1	WIND2	ENOISE	ANOISE
SHRI	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
BMOMI	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
TORI	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
SHRO	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
BMOMO	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
TORO	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
ELD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
EL	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILOD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
AILO	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

F

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
ELEVC	1.5832E-04	-2.3582E-04	1.1387E-04	1.8312E-04	-4.1426E-05	4.8678E-05	2.6735E-04	1.7564E-04	-9.1525E-06	-1.9235E-05
AILC	3.0315E-03	2.6654E-03	4.1081E-04	7.8271E-04	1.4589E-04	-8.5377E-04	-1.2540E-03	4.6786E-04	1.3813E-06	-6.7161E-05

	X11	X12	X13	X14	X15	X16	X17	X18	X19	X20
ELEVC	2.1995E-04	-4.5889E-05	-1.7184E-06	-4.3851E-05	-8.0336E-05	-2.5128E-05	-6.4003E-06	9.7488E-06	-2.4859E-06	1.1835E-05
AILC	9.5182E-04	3.5894E-04	6.2767E-05	3.7742E-04	-9.2428E-05	6.5354E-04	4.6382E-05	-2.8777E-04	-1.6007E-04	-1.7710E-04

	X21	X22	X23	X24	X25	X26	X27	X28	X29	X30
ELEVC	-3.6349E-06	7.3601E-06	-1.4385E-05	-8.0966E-06	-2.9199E-05	7.3229E-06	2.0059E-05	1.9455E-06	-5.0617E-05	-1.4932E-05
AILC	2.4929E-04	8.8603E-05	3.1912E-04	2.5139E-04	-1.1683E-04	-9.9825E-04	2.7410E-04	7.8487E-04	-9.1330E-04	-6.2633E-04

	X31	X32	X33	X34	X35	X36	X37	X38	X39	X40
ELEVC	2.9402E-05	6.3263E-06	2.3606E-06	-7.1657E-05	1.0550E-05	-2.3694E-08	6.8456E-08	1.2641E-08	3.0596E-10	4.0774E-07
AILC	5.0017E-04	-5.6772E-05	1.5107E-05	1.5403E-03	1.0277E-03	-9.3928E-08	-5.9855E-07	-3.5760E-07	3.5076E-07	5.9781E-06

	X41	X42	X43	X44	X45	ELEV	ELEVD	ELEVDD	AIL	AILD
ELEVC	-2.9662E-07	9.0481E-10	-5.4855E-10	3.9594E-06	1.6946E-09	-0.4132	-6.8441E-03	-6.4513E-06	-2.1111E-02	-2.7671E-04
AILC	6.4990E-06	-2.7729E-09	5.3934E-09	-3.8792E-04	-3.7901E-08	-0.6329	-7.2630E-03	-7.0158E-06	-1.360	-2.1233E-02

	AILDD	WIND1	WIND2	ENOISE	ANOISE
ELEVC	-2.8063E-07	8.0010E-05	1.0842E-04	3.541	0.1928
AILC	-2.0544E-05	2.2294E-04	1.4034E-03	2.677	8.327

L

	QCG	WTIPDD
X1	1785.	2.272
X2	3319.	2.6873E-02
X3	70.88	2.3472E-02
X4	554.4	2.5272E-02
X5	-689.6	-0.2266
X6	-43.74	0.1884
X7	-39.38	-9.2332E-02
X8	-108.3	4.5919E-02
X9	-105.6	0.2794
X10	777.9	0.2092
X11	1.8692E+04	4.8515E-02
X12	-9112.	-1.195
X13	2483.	-0.3082
X14	8.407	-2.693
X15	3332.	4.719
X16	6892.	0.1216
X17	189.0	0.1776
X18	-14.07	-0.3136
X19	-698.0	-0.6674
X20	960.5	-0.7365
X21	90.55	0.2265
X22	-6.130	9.7791E-02
X23	306.2	0.1553
X24	44.03	5.8746E-02
X25	-2799.	-5.226
X26	634.2	0.6398
X27	1756.	-1.150
X28	3578.	-0.9081
X29	3122.	0.3963
X30	13.31	-2.7236E-02
X31	-30.99	0.1238
X32	6683.	1.259
X33	-8069.	-0.6234
X34	-7828.	0.6847
X35	8314.	4.343
X36	86.05	1.0999E-02
X37	-20.95	9.2423E-02
X38	43.47	-9.7774E-02
X39	82.23	5.1789E-02
X40	-296.3	-0.2155
X41	60.50	1.2172E-02
X42	-20.62	-9.3657E-03
X43	18.70	4.0876E-04
X44	7830.	3.815
X45	2354.	1.587
ELEV	0.8878	-4.1163E-06
ELEVD	11.43	-1.2973E-03
ELEVDD	380.9	1.298
AIL	-3.558	1.2771E-03
AILD	-98.84	-0.5599
AILDD	1.7817E+04	40.63
WIND1	-4600.	-2.597
WIND2	-1.3619E+04	-8.357
ENOISE	-3.9461E-02	3.3987E-08
ANOISE	-0.3878	-3.7938E-04

Servo-Control

Problem #90-07 HYDRAULIC POSITIONING SYSTEM

(A) General Description

It is desired to control the carriage position of a hydraulic positioning system under different loading conditions, for a linear plant model which is subject to large uncertainty.

(B) Reference

(a) Gu Qi-tai, "A Robust Regulator for a Hydraulic Positioning System", *Technische Hogeschool Twente Memorandum*, no. 339.

(b) Baumgartner, H., "Die Nachlaufgenauigkeit des Hydraulischen Zylinderantriebes im Maschinenbau", Diss. ETH 5963, Zürich 1977.

(c) Senning, M., "Robuste Regelung Hydraulischer Vorschubantriebe", Vortrag Seminar Inst. f. Werkzeugmaschinenbau und Fertigungstechnik, ETH-Z, Zürich, 1980.

(C) Problem Description

A model of a SISO hydraulic positioning system is given as follows:

$$\dot{x} = A(p)x + b(p)u + e(p)\omega$$
$$y = c(p)x$$

where $x \in R^3$, $u \in R^1$ is the input, $y \in R^1$ is the output to be regulated, $\omega \in R^1$ is a disturbance, and the plant model is parametrized by parameters $p \in R^8$, which lie in specified bounds $p \in \mathcal{P}$.

- x_1 = output position of carriage (cm) (y in Figure 1)
- x_2 = velocity of carriage = $\dot{x}_1$ (cm/sec) ($\dot{y}$ in Figure 1)
- x_3 = oil pressure (P_L in Figure 1)
- u = servo valve flow input
- y = output position of carriage to be regulated (y in Figure 1)
- ω = external force disturbance (F_L in Figure 1, $0 < \|F_L\| < 800 d_a N$)

It is desired to find a controller to regulate the output y to a desired set point tracking signal, subject to the following constraints:

(a) The output response has zero overshoot for a step function set point change, or/and a step function disturbance change in ω.

(b) The output has a zero trailing error for ramp shaped reference input signal.

(c) $\|u\| \leq 866$ cm^3/sec

(d) The above constraints should be satisfied for all parameters $p \in \mathcal{P}$.

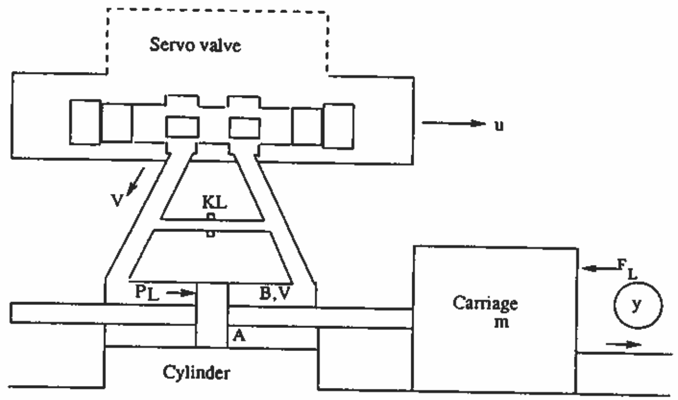

Figure 1: Schematic representation of hydraulic positioning system

The data for this problem is given as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\left(R + \frac{4}{\pi}R_c\right)/m & A/m \\ 0 & -\frac{4B}{V}A & -\frac{4B}{V}(\sigma + K_L) \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ \frac{4B}{V} \end{bmatrix}, \quad e = \begin{bmatrix} 0 \\ -1/m \\ 0 \end{bmatrix}$$

$$c = [1 \ \ 0 \ \ 0]$$

where the parameters B, m, R, R_c, K_L, σ, V, A are given as follows:

B	compressibility modulus (nominal value $B = 14000$) ;	$9000 < B < 16000$ daN/cm^2
m	mass (nominal value $m = 0.1287$),	$0.05 < m < 0.3$ t
R	friction coefficient (nominal value $R = 0.150$)	$0.05 < R < 5$ daN/cm
R_c	Coulomb friction coefficient (nominal $R_c = 0.01$)	$0 < R_c < 0.05$ s daN/cm
K_L	bypass oil coefficient (nominal $K_L = 0.002$)	$0.000103 < K_L < 0.0035$ cm^5/s daN
σ	hydraulic constant (nominal $\sigma = 0.24$)	$0.001 < \sigma < 15$ cm^5/s daN
V	volume (nominal value $V = 874$ cm^3)	
A	piston area (nominal value $A = 10.75$),	$10.5 < A < 11.10$ cm^2
F_L	external force	$0 < F_L < 800$ daN

(D) Name of Person Submitting Problem

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Problem #90-08 CONTROL SURFACE SERVO PROBLEM FOR AN UNDERWATER VEHICLE

(A) General Description

We consider in this problem a control surface servo for an underwater vehicle. The system is a 2 input, 3 output system with disturbances. The main features of interest are:

1. A lightly damped resonant load mode with characteristics dependent on speed and depth.
2. A disturbance-rejection specification which depends on disturbance frequency.
3. A parameter sensitivity specification.
4. Strict limitations on the complexity of the feedback structure, i.e. the controller is constrained to have a decentralized control structure.

(B) Reference

This problem has not been published previously. Permission of the Royal Canadian Navy to publish it is gratefully acknowledged.

(C) Problem Description

System Overview

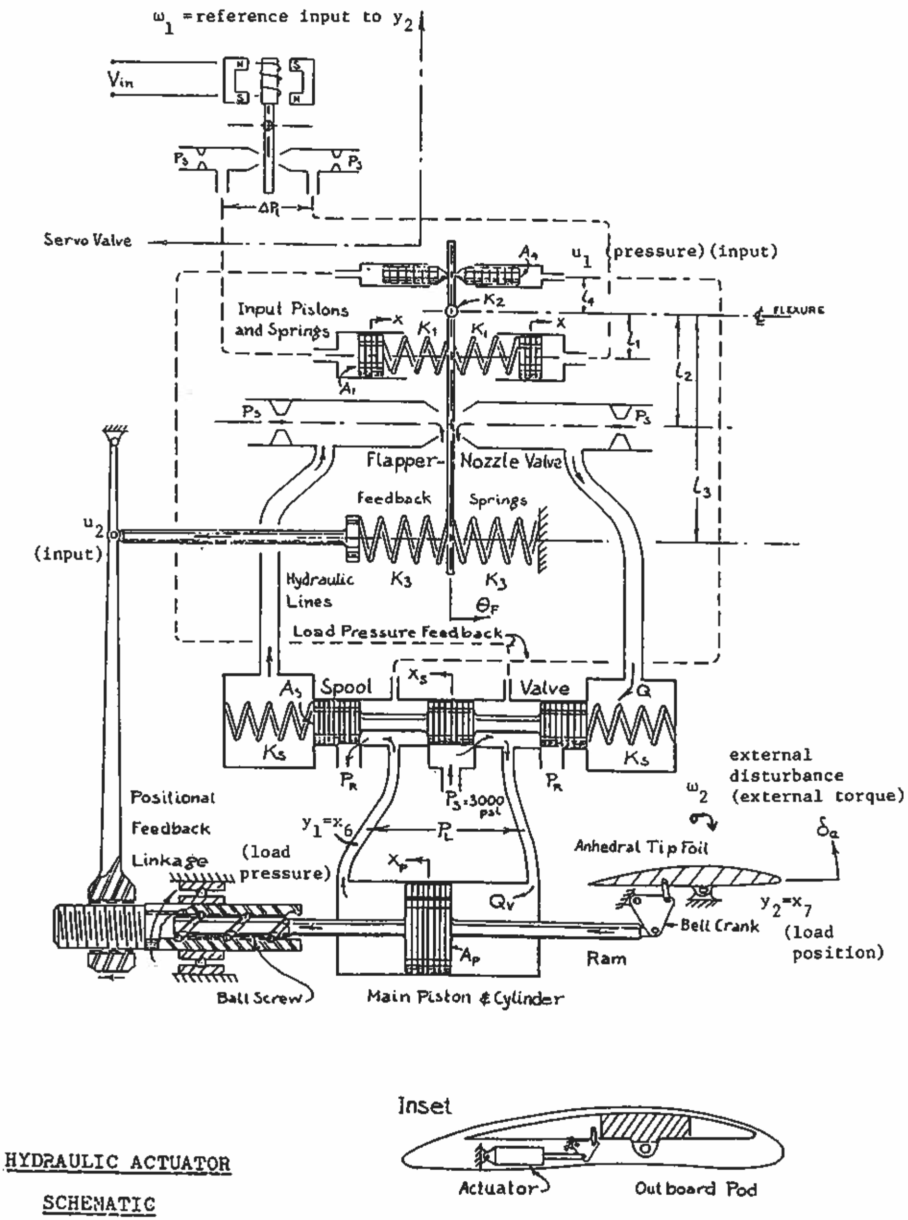
The servo, which is completely hydraulic, is shown in Figure 1. The input differential pressure, ΔP_i, is derived from an electrohydraulic valve with negligible dynamics. Two stages of amplification follow. The flapper valve acts as a force summer, having as input forces:

1. An input force proportional to ΔP_i, applied through pistons;
2. A restoring force provided by a flexural spring;
3. A position feedback feedback force, applied by an adjustable feedback linkage through pistons and springs;
4. A load pressure feedback force, applied through small pistons.

System Model

State, control and output variables

The mass of the input pistons is ignored, and the input force is assumed to be applied directly to the flapper. Inertia effects in hydraulic lines are also ignored, since the lines are very short.



HYDRAULIC ACTUATOR

SCHEMATIC

Fig. 1

With these simplifications, the system can be modelled as an 8th order system. The system equations have the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{E}\mathbf{w}$$
$$\mathbf{y}_m = \mathbf{C}_m\mathbf{x}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x}$$

The state vector

$$\mathbf{x} = (x_1, \cdots, x_8)'$$

where

x_1 = flapper valve position
x_2 = flapper valve velocity
x_3 = flapper output pressure
x_4 = spool valve position
x_5 = spool valve velocity
x_6 = load pressure
x_7 = load position
x_8 = load velocity

The control vector, which represents the two feedback connections, is

$$\mathbf{u} = (u_1, u_2)'$$

where

u_1 = load pressure feedback input
u_2 = position feedback input

The disturbance vector incorporates both the input signal and the load disturbance hydrodynamic force referred to the ram;

$$\mathbf{w} = (w_1, w_2)'$$

where

w_1 = input signal
w_2 = load force (external disturbance)

The measurable output variables are:

$$\mathbf{y}_m = (x_6, x_7)'$$

and the output to be regulated is:

$$y = x_7$$

The state variables are normalized, with 1 representing the saturation limit of each variable. The output variables are therefore also normalized, with 1 representing the limits of the linear range of the model. The w vector is normalized so that 1 represents 5 volts (electrical input) for w_1, and 50,000 lb for w_2.

The gain (k_2) of the feedback linkage is adjustable from a ratio R_f of 0.04 to R_f=0.30. The load pressure feedback gain (k_1) is adjustable from 0 to 360. The feedback configuration is thus constrained, to have the structure:

$$u = \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} y_m$$

where k_1, k_2 are constant gains to be determined.

The load is a control surface whose effective mass (referred to the ram), effective damping force, and hydrodynamic/structural spring constant vary with vehicle speed and immersion depth. A single set of feedback gains is sought which will satisfy the system specifications over the operating range.

Matrix elements

0	8.5e+02	0	0	0	0	0	0
-8.5e+02	-1.2e+02	-4.1e+03	0	0	0	0	0
3.3e+01	0	-3.3e+01	0	-7.0e+02	0	0	0
0	0	0	0	1.4e+03	0	0	0
0	0	1.6e+03	-4.5e+02	-1.1e+02	0	0	0
0	0	0	8.1e+01	0	-1.0e+00	0	-9.0e+02
0	0	0	0	0	0	0	1.1e+02
0	0	0	0	0	1.2e+01	-1.1e+00	-2.2e+01

Table 1: A matrix

0	0
4.6e+00	9.9e+04
0	0
0	0
0	0
0	0
0	0
0	0

Table 2: B matrix

0	0
9.9e+0.3	0
0	0
0	0
0	0
0	0
0	0
0	9.9e+0.1

Table 3: E matrix

$$C_m = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$
$$C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Table 4: C_m and C matrices

Load characteristics The hydrodynamic load on the control surface is speed and depth dependent. This dependence can be expressed in terms of a single parameter p. Over the operating range, p varies from 0.5 to 1. The values given for the A matrix in Table 1 are for $p = 1$. For other values of p, the values of the matrix elements which vary are given by:

$$\begin{aligned} A(8,6) &= \frac{A(8,6)_{p=1}}{2.48 - 1.48p} \\ A(8,7) &= F(p)A(8,7)_{p=1} \\ A(8,8) &= F(p)A(8,8)_{p=1} \\ E(8,2) &= \frac{E(8,2)_{p=1}}{2.48 - 1.48p} \end{aligned}$$

where

$$F(p) = \frac{0.8065p^2(1.5 - p)}{1 - 0.5968p}$$

Performance Specifications

(a) Response to command inputs: For a step input of 0.25 unit at w_1, the settling time for y shall be less than 0.1 s and the peak overshoot for y shall be less than 20 percent. No state variable shall reach saturation for this input level. These requirements are to be satisfied for all operating conditions, i.e. $\forall p \in [0.5, 1]$.

(b) Stiffness requirement: The steady-state sinusoidal frequency response $g(j\omega) \triangleq y(j\omega)/w_2(j\omega)$ shall have the property that its steady state gain $\|g(0)\|$ shall not exceed 1 unit, its maximum gain $g_{max} \triangleq \max_{\omega\in[0,\infty)} \|g(j\omega)\|$ should not exceed 3 times its steady-state gain $\|g(0)\|$, and $| g(j\omega) | < g_{max}$, $\forall$ frequencies ≥ 8 Hz. (This specification is required to preclude hydrodynamically induced structural divergence, and is mandatory.) These requirements are to be satisfied for all operating conditions, i.e. $\forall p \in [0.5, 1]$.

(c) Sensitivity specifications: All zero entries in the system matrices, and the elements A(1,2) and A(7,8), are fixed. The remaining elements may vary by ± 20 per cent. Stiffness requirements (b) are to be met for all such variations, and stability is to be retained. *Minor* variations in input-output response (specifications a) can be tolerated in this case.

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Mechanical Systems

Problem #90-09 (Cascade of Inverted Pendula)
Problem #90-10 (Regulation of a Ship's Heading)
Problem #90-11 (Automatic track Control of City Bus)
Problem #90-12 (Robotic System)

Mechanical

Problem #90-09 CASCADE OF INVERTED PENDULA

(A) General Description

This problem describes a linearized model of a cascade of inverted pendula, and has the feature of being 'highly' unstable and difficult to control. The difficulties of control become more pronounced as the number of links increase.

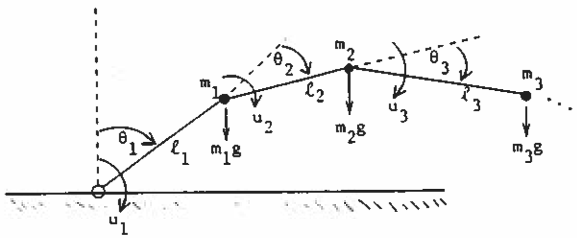
(B) Reference

The source of this problem is given in:

Kwakernaak, H., Westdyk, H., "Regulability of a multiple inverted pendulum system", *Control-Theory and Advanced Technology*, vol. 1, no. 1, April 1985, pp. 1-9.

(C) Problem Description

Consider the following system of a cascade of inverted pendula:



where all point masses $m_i = 1$ (kg), all links have length $l_i = 1$ (m), $g = 9.8$ (m/sec^2), and where $u_1, u_2, u_3, \ldots$ denote torques about the respective pivots. Let the outputs of the system be $y_i = \theta_i$, $i = 1, 2, 3, \ldots$, and the inputs to the system be u_i, $i = 1, 2, 3, \ldots$.

It is desired to design a controller to stabilize the system so that the outputs are regulated to zero, in the presence of unmeasurable constant disturbances which may be applied to the system.

The following linearized model:

$$\dot{x} = a_i x + b_i u$$
$$y = c_i x$$

describes the system with i links, $i = 1, 2, 3, 4, 5, 6, 10$:

```
a1 =

           0   1.0000e+00
  9.8000e+00            0

b1 =

     0
     1

c1 =

     1     0

a2 =

           0   1.0000e+00            0            0
  9.8000e+00            0  -9.8000e+00            0
           0            0            0   1.0000e+00
 -9.8000e+00            0   2.9400e+01            0

b2 =

     0     0
     1    -2
     0     0
    -2     5

c2 =

     1     0     0     0
     0     0     1     0

a3 =

           0            0            0   1.0000e+00            0            0
           0            0            0            0   1.0000e+00            0
           0            0            0            0            0   1.0000e+00
  2.9400e+01  -1.9600e+01  -3.6285e-16            0            0            0
 -2.9400e+01   3.9200e+01  -9.8000e+00            0            0            0
           0  -1.9600e+01   1.9600e+01            0            0            0

b3 =

           0            0            0
           0            0            0
           0            0            0
  1.6667e+00  -2.6667e+00   1.0000e+00
 -2.3333e+00   4.8333e+00  -3.5000e+00
  6.6667e-01  -2.6667e+00   4.0000e+00

c3 =

     1     0     0     0     0     0
```

```
     0     1     0     0     0     0
     0     0     1     0     0     0

a4 =

  Columns 1 through 6

            0            0            0            0   1.0000e+00            0
            0            0            0            0            0   1.0000e+00
            0            0            0            0            0            0
            0            0            0            0            0            0
   3.9200e+01  -2.9400e+01            0            0            0            0
  -3.9200e+01   5.8800e+01  -1.9600e+01  -3.6285e-16            0            0
  -4.3521e-15  -2.9400e+01   3.9200e+01  -9.8000e+00            0            0
            0            0  -1.9600e+01   1.9600e+01            0            0

  Columns 7 through 8

            0            0
            0            0
   1.0000e+00            0
            0   1.0000e+00
            0            0
            0            0
            0            0
            0            0

b4 =

            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
   1.7500e+00  -2.7500e+00   1.0000e+00   6.4185e-17
  -2.5000e+00   5.1667e+00  -3.6667e+00   1.0000e+00
   7.5000e-01  -3.0833e+00   4.8333e+00  -3.5000e+00
   3.5586e-17   6.6667e-01  -2.6667e+00   4.0000e+00

c4 =

     1     0     0     0     0     0     0     0
     0     1     0     0     0     0     0     0
     0     0     1     0     0     0     0     0
     0     0     0     1     0     0     0     0

a5 =

  Columns 1 through 6

            0            0            0            0            0   1.0000e+00
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
   4.9000e+01  -3.9200e+01  -4.5686e-15   1.0878e-15            0            0
  -4.9000e+01   7.8400e+01  -2.9400e+01            0            0            0
  -3.6630e-14  -3.9200e+01   5.8800e+01  -1.9600e+01  -3.6285e-16            0
   1.2240e-14  -1.3056e-14  -2.9400e+01   3.9200e+01  -9.8000e+00            0
            0            0            0  -1.9600e+01   1.9600e+01            0

  Columns 7 through 10

            0            0            0            0
   1.0000e+00            0            0            0
            0   1.0000e+00            0            0
            0            0   1.0000e+00            0
            0            0            0   1.0000e+00
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0

b5 =

            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
   1.8000e+00  -2.8000e+00   1.0000e+00   3.1160e-16  -2.2755e-16
  -2.6000e+00   5.3500e+00  -3.7500e+00   1.0000e+00   3.0015e-16
   8.0000e-01  -3.3000e+00   5.1667e+00  -3.6667e+00   1.0000e+00
   5.1282e-16   7.5000e-01  -3.0833e+00   4.8333e+00  -3.5000e+00
   5.3006e-17  -1.2003e-17   6.6667e-01  -2.6667e+00   4.0000e+00

c5 =

     1     0     0     0     0     0     0     0     0     0
     0     1     0     0     0     0     0     0     0     0
     0     0     1     0     0     0     0     0     0     0
     0     0     0     1     0     0     0     0     0     0
     0     0     0     0     1     0     0     0     0     0

a6 =

  Columns 1 through 6

            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
   5.8800e+01  -4.9000e+01   2.9028e-15  -1.6328e-15   5.4427e-16  -1.0086e-32
  -5.8800e+01   9.8000e+01  -3.9200e+01  -4.8961e-15   5.4401e-16   2.0127e-32
            0  -4.9000e+01   7.8400e+01  -2.9400e+01            0            0
  -2.0310e-14   2.0491e-14  -3.9200e+01   5.8800e+01  -1.9600e+01  -3.6285e-16
   1.1968e-14  -1.5504e-14   5.4401e-15  -2.9400e+01   3.9200e+01  -9.8000e+00
   2.1755e-15  -1.6323e-15   2.1760e-15  -1.0885e-15  -1.9600e+01   1.9600e+01

  Columns 7 through 12

   1.0000e+00            0            0            0            0            0
            0   1.0000e+00            0            0            0            0
            0            0   1.0000e+00            0            0            0
            0            0            0   1.0000e+00            0            0
```

```
         0          0          0          0 1.0000e+00          0
         0          0          0          0          0 1.0000e+00
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0

b6 =

         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
1.8333e+00 -2.8333e+00 1.0000e+00 -3.0442e-16 1.2911e-16 -1.0963e-16
-2.6667e+00 5.4667e+00 -3.8000e+00 1.0000e+00 6.7758e-17 4.2643e-16
8.3333e-01 -3.4333e+00 5.3500e+00 -3.7500e+00 1.0000e+00 -7.5488e-16
-5.5618e-16 8.0000e-01 -3.3000e+00 5.1667e+00 -3.6667e+00 1.0000e+00
4.3692e-16 -9.2492e-16 7.5000e-01 -3.0833e+00 4.8333e+00 -3.5000e+00
1.1031e-16 -1.6825e-16 2.2085e-17 6.6667e-01 -2.6667e+00 4.0000e+00

c6 =

  1  0  0  0  0  0  0  0  0  0  0  0
  0  1  0  0  0  0  0  0  0  0  0  0
  0  0  1  0  0  0  0  0  0  0  0  0
  0  0  0  1  0  0  0  0  0  0  0  0
  0  0  0  0  1  0  0  0  0  0  0  0
  0  0  0  0  0  1  0  0  0  0  0  0

a10 =

 Columns 1 through 6

         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
9.8000e+01 -8.8200e+01 -1.7383e-15 -3.7188e-18 -6.5249e-15 5.4388e-15
-9.8000e+01 1.7640e+02 -7.8400e+01          0          0          0
-1.0880e-14 -8.8200e+01 1.5680e+02 -6.8600e+01 6.5281e-15 -1.0880e-14
1.5855e-14 -3.6370e-14 -7.8400e+01 1.3720e+02 -5.8800e+01 3.4194e-14
-1.1062e-13 1.0699e-13 -2.5751e-14 -6.8600e+01 1.1760e+02 -4.9000e+01
1.0880e-13 -9.7922e-14 1.3056e-14 -7.6177e-15 -5.8800e+01 9.8000e+01
-8.1596e-15 6.5289e-15 -8.1601e-15 -5.4454e-16 3.1009e-14 -4.9000e+01
-4.9868e-14 4.2342e-14 3.9892e-15 -1.4506e-15 -1.8859e-14 2.9013e-14
2.7200e-14 -2.3256e-14          0          0 8.1601e-15 -9.5202e-15
         0          0          0          0          0          0

 Columns 7 through 12

         0          0          0          0 1.0000e+00          0
         0          0          0          0          0 1.0000e+00
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
1.3084e-31 -1.6316e-15 5.4388e-16 -1.0042e-32          0          0
         0          0          0          0          0          0
4.3521e-15 1.4495e-30 -4.8318e-31 8.9449e-48          0          0
-1.9585e-14 3.2649e-15 -1.0883e-15 2.0172e-32          0          0
1.5959e-14 -4.8969e-15 1.6323e-15 4.0225e-32          0          0
-3.9200e+01 -1.3056e-14 4.3521e-15 -8.0569e-32          0          0
7.8400e+01 -2.9400e+01 -4.8961e-15 8.0569e-32          0          0
-3.9200e+01 5.8800e+01 -1.9600e+01 3.6285e-16          0          0
-4.3521e-15 -2.9400e+01 3.9200e+01 -9.8000e+00          0          0
         0          0 -1.9600e+01 1.9600e+01          0          0

 Columns 13 through 18

         0          0          0          0          0          0
         0          0          0          0          0          0
1.0000e+00          0          0          0          0          0
         0 1.0000e+00          0          0          0          0
         0          0 1.0000e+00          0          0          0
         0          0          0 1.0000e+00          0          0
         0          0          0          0 1.0000e+00          0
         0          0          0          0          0 1.0000e+00
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0

 Columns 19 through 20

         0          0
         0          0
         0          0
         0          0
         0          0
         0          0
         0          0
         0          0
1.0000e+00          0
         0 1.0000e+00
         0          0
         0          0
         0          0
         0          0
         0          0
```

```
         0         0
         0         0
         0         0
         0         0
         0         0
```

b10 =

Columns 1 through 6

```
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
  1.9000e+00  -2.9000e+00   1.0000e+00   2.1959e-16  -2.8176e-16   4.8626e-16
 -2.8000e+00   5.6889e+00  -3.8889e+00   1.0000e+00  -8.3224e-16   3.5204e-16
  9.0000e-01  -3.6778e+00   5.6528e+00  -3.8750e+00   1.0000e+00  -2.5226e-15
 -5.1230e-16   8.8889e-01  -3.6389e+00   5.6071e+00  -3.8571e+00   1.0000e+00
 -2.3590e-15   3.7146e-15   8.7500e-01  -3.5893e+00   5.5476e+00  -3.8333e+00
  2.6600e-15  -4.0482e-15   2.3538e-15   8.5714e-01  -3.5238e+00   5.4667e+00
 -5.6230e-16   8.7784e-16  -5.4097e-16  -7.1966e-16   8.3333e-01  -3.4333e+00
 -8.0494e-16   1.1525e-15  -2.2138e-16   3.3885e-16  -1.0433e-15   8.0000e-01
  5.2901e-16  -7.6095e-16   1.7267e-16  -1.0799e-16   4.5900e-16  -1.2346e-16
 -3.7739e-17   1.5780e-17   5.5573e-17  -9.0729e-18  -4.6091e-17   7.8485e-17
```

Columns 7 through 10

```
           0            0            0            0
           0            0            0            0
           0            0            0            0
           0            0            0            0
           0            0            0            0
           0            0            0            0
           0            0            0            0
           0            0            0            0
           0            0            0            0
           0            0            0            0
 -1.0559e-16  -2.5389e-16   7.9671e-17   3.8476e-17
 -3.3739e-16   5.2418e-16   8.8172e-17  -2.6452e-16
  1.5710e-15  -8.8962e-16  -9.7319e-17   2.9196e-16
 -3.6967e-15   1.8224e-15  -2.0082e-16   4.7207e-17
  1.0000e+00  -2.3237e-15   4.6098e-16  -5.5013e-16
 -3.8000e+00   1.0000e+00   2.8896e-16   1.0290e-15
  5.3500e+00  -3.7500e+00   1.0000e+00  -1.6135e-15
 -3.3000e+00   5.1667e+00  -3.6667e+00   1.0000e+00
  7.5000e-01  -3.0833e+00   4.8333e+00  -3.5000e+00
 -2.2627e-16   6.6667e-01  -2.6667e+00   4.0000e+00
```

c10 =

Columns 1 through 12

```
     1     0     0     0     0     0     0     0     0     0     0     0
     0     1     0     0     0     0     0     0     0     0     0     0
     0     0     1     0     0     0     0     0     0     0     0     0
     0     0     0     1     0     0     0     0     0     0     0     0
     0     0     0     0     1     0     0     0     0     0     0     0
     0     0     0     0     0     1     0     0     0     0     0     0
     0     0     0     0     0     0     1     0     0     0     0     0
     0     0     0     0     0     0     0     1     0     0     0     0
     0     0     0     0     0     0     0     0     1     0     0     0
     0     0     0     0     0     0     0     0     0     1     0     0
```

Columns 13 through 20

```
     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0
```

(D) Name of Person Submitting Problem

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Problem #90-10 REGULATION OF A SHIP'S HEADING

(A) General Description

This problem deals with the design of a controller associated with the regulation of a ship's heading, in which constraints are imposed on the magnitude of the control signals applied.

(B) Reference

The source of this problem is given in:

Kallstrom, C.G., K.J. Astrom, N.E. Thorell, J. Eriksson, L. Sten, "Adaptive Autopilots for Tankers", *Automatica*, vol. 15, no. 3, 1979, pp. 241-254.

Astrom, K.J., Kallstrom, C.G., "Identification of Ship Steering Dynamics", *Automatica*, vol. 12, 1976, pp. 9-22.

Kallstrom, C.G., Astrom, K.J., "Experiences of System Identification Applied to Ship Steering", *Automatica*, vol. 17, no. 1, 1981, pp. 187-198.

(C) Problem Description

The linearized model of a ship moving under constant velocity (in a straight line motion) is described as follows:

$$\dot{x} = Ax + bu$$
$$y = cx$$

where $x \in R^3$, $u \in R^1$, $y \in R^1$ are given as follows:

u = rudder angle (radians) (δ in Figure 1)
y = heading angle of ship (radians) (ψ in Figure 1)
x_1 = sway velocity of ship (v in Figure 1)
x_2 = turning yaw rate ($r = \dot{\psi}$ in Figure 1)
x_3 = heading angle of ship (radians) (ψ in Figure 1)

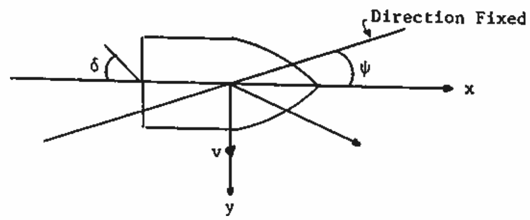

Figure 1: Notation used to describe ship's motion

It is desired to find a controller for the system to regulate the heading angle of the ship to a desired angle y_{ref}, such that the following constraints are satisfied:

(a) No overshoot occurs in the output response of y.

(b) The rudder motion is constrained:

$$\|u\| \leq 40^\circ$$

(c) The rate of rudder motion is constrained:

$$\|\dot{u}\| \leq 10^\circ/\text{sec}$$

The turning rate $\dot{\psi}$ of the vessel may be measured, and used in the controller. The data for this problem is given in normalized units as follows:

length unit = length of ship

time unit = time required for ship to travel a ship's length

where the structure of A, b, c is given by:

$$A = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} b_1 \\ b_2 \\ 0 \end{pmatrix}, \quad c = (0 \;\; 0 \;\; 1)$$

where a_{11}, a_{12}, a_{22}, b_1, b_2 is given as follows for various classes of vessels:

vessel	Cargo #1	Cargo #2	Tanker #1	Tanker #2	Tanker #3
length	160	161	305	322	350
speed	15.2	15	16	16	15.8
a_{11}	-0.895	-0.770	-0.597	-0.298	-0.454
a_{12}	-0.286	-0.335	-0.372	-0.279	-0.433
a_{21}	-4.367	-3.394	-3.651	-4.370	-4.005
a_{22}	-0.918	-1.627	-0.792	-0.773	-0.807
b_1	0.108	0.170	0.103	0.116	0.097
b_2	-0.918	-1.627	-0.792	-0.773	-0.807

If one wishes to explore a nonlinear model, the first equation can be modified to give

$$\frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + cx_1|x_1| + b_1u$$

where $c = 0.7$.

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Problem #90-11 AUTOMATIC TRACK CONTROL OF CITY BUS

(A) General Description

It is desired to control the lateral motion of a city bus such that the bus follows an electromagnetic track imbedded in the street. Uncertain parameters of the system are the bus's mass, the bus's speed and road surface conditions.

(B) References

The source of this problem is given in:

Darenberg, W., "Automatische Spurfuehrung von Kraftfahrzeugen", *Automobilindustrie*, 1987, pp. 155-159.

Ackermann, J., "The Track-Guided Bus - Parametric Modelling of its Dynamics and Analysis of the Robust Control Problem", DLR Report 515-89-15, July 1989.

(C) Problem Description

A linearized model for the lateral motion of a city bus is given as follows:

$$\dot{x} = A(q)x + bu$$
$$y = Cx$$

where $x \in R^5$, $u \in R^1$ the input, $y \in R^1$ the output to be regulated, and the plant model, parametrized by parameters $q \in R^2$, which lie in specified bounds $q \in Q$. The variables of the model are:

x_1 = sideslip angle at center of gravity (α in Figure 1)
x_2 = yaw angle rate ($\dot{\varepsilon}$ in Figure 1)
x_3 = deviation of center of bus from track (d in Figure 1)
x_4 = yaw angle (ε in Figure 1)
x_5 = steering angle (radians) (β in Figure 1)
u = input to integrating hydraulic steering actuator ($\dot{\beta}$ in Figure 1)
y = displacement of front sensor of bus from track
q_1 = reciprocal of bus velocity ($1/v$ in Figure 1)
q_2 = ratio of friction coefficient and mass of bus ($q_2 = \mu/m$)

A, b, C are given as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & a_{15} \\ a_{21} & a_{22} & 0 & 0 & a_{25} \\ 1/q_1 & 0 & 0 & 1/q_1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$
$$C = [0 \ \ 0 \ \ 1 \ \ l_S \ \ 0]$$

$a_{11} = -2q_1q_2(\delta_F + \delta_R)$ $\quad a_{12} = -1 - 2q_1^2q_2(\delta_F l_F - \delta_R l_R)$
$a_{21} = -2q_2(\delta_F l_F - \delta_R l_R)/c$ $\quad a_{22} = -2q_1q_2(\delta_F l_F^2 + \delta_R l_R^2)/c$
$a_{15} = 2q_1q_2\delta_F$
$a_{25} = 2q_2\delta_F l_F/c$

where the constant parameters of the vehicle are given by:

l_S = distance of front sensor from center of mass = 6.12 m
l_F = distance of front axle from center of mass = 3.67 m
l_R = distance of rear axle from center of mass = 1.93 m
δ_F = lateral force coefficient front wheels ($\mu = 1$) = 99 kN/rad
δ_R = lateral force coefficient rear wheels ($\mu = 1$) = 235 kN/rad
c = ratio θ/m, where θ is the moment of inertia w.r.t. a vertical axis through the center of mass and m is the bus mass, $c = 10.86$ m^2

The uncertain operating conditions of the vehicle are given as follows:

$$0.5 \text{ (wet road)} \le \mu \le 1 \text{ (dry road)}$$
$$9.95 \text{ tons} \le m \le 16 \text{ tons}$$
$$1ms^{-1} \le v \le 20ms^{-1}$$

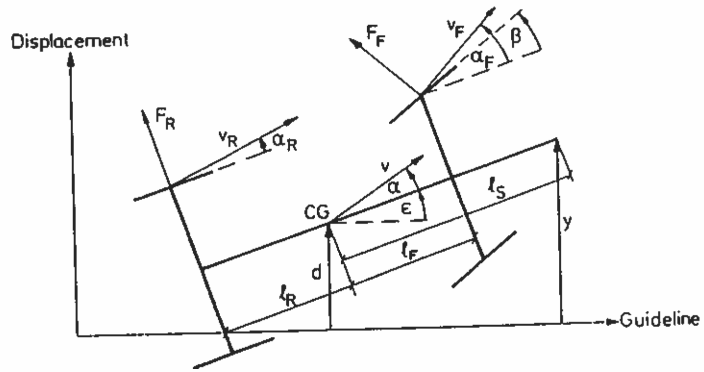

Figure 1: Automatic steering of city bus

Let y_{Track} be the displacement of the track guideline from a given reference; then $\ddot{y}_{Track} = v\omega_{ref}$, where ω_{ref} depends on the geometry of the track guideline. In a curve, the lateral acceleration of the bus at the front sensor ($\ddot{y}_{Bus}$) is given by $\ddot{y}_{Bus} = v\omega$, where $\omega := \dot{\varepsilon} + \dot{\alpha} + l_s\frac{\ddot{\varepsilon}}{v}$, and thus the second derivative of the measured displacement ($\ddot{e}$) is given by $\ddot{e} = \ddot{y}_{Bus} - \ddot{y}_{Track}$. Thus if $\omega_{ref} \equiv 0$ (i.e. the track guideline is a straight line), then $\ddot{e} = \ddot{y}_{Bus} = v\omega = v(\dot{\varepsilon} + \dot{\alpha}) + l_s\ddot{\varepsilon}$ which gives $e = d + l_s\varepsilon = y$. If $\omega_{ref} \not\equiv 0$, then the error e in the system, to be regulated to zero, is given by $\ddot{e} = v(\omega - \omega_{ref})$ or by

$$e = y - y_{track}$$

where $\ddot{y}_{track} = v\omega_{ref}$.

Typical transition trajectories are:

- Switching from manual to automatic steering. In this case, the controller is activated before switching with initial condition $x_1(0) = 0$, $x_2(0) = 0$, $x_3(0) = 0.15$, $x_4(0) = 0$, $x_5(0) = 0$, which corresponds to a 15 cm parallel displacement from track.
- Transition from a straight track into a curve of constant radius 400 m.
- Narrow S-shaped curve for entering a bus stop bay at low speed with ω_{ref} given in Figure 2.

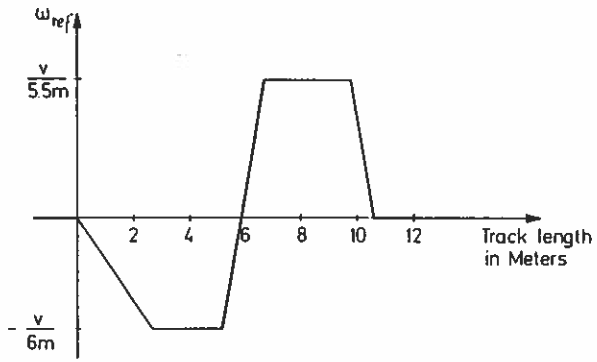

Figure 2: Reference angular velocity for entering a narrow bus stop bay

The following constraints must be satisfied during the above transitions:

For passenger comfort the natural frequency ω_n of the lateral motion must not exceed 1.2 Hz, i.e.

$$\omega_n \leq 2\pi \cdot 1.2 = 7.5 \text{ Radians/sec}$$

The lateral acceleration must not exceed $0.4g$, i.e.

$$v(\omega - \omega_{ref}) \leq 4ms^{-2}$$

The steering angle is limited by

$$|\beta| \leq 40°$$

and its rate by

$$|\dot{\beta}| \leq 23°/\text{sec}$$

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Mechanical

Problem #90-12 ROBOTIC SYSTEM

1. Introduction.

Control of robots is a topic of current interest. However, most control strategies neglect actuator dynamics. There are two main approaches suggested in the literature (see e.g. reference list in [1]). The first is based only on the geometric equations relating task space coordinates and configuration space coordinates. The second one applies classical (e.g. PID) or modern (e.g. MRAC, optimality concepts, sliding modes etc.) control concepts to the so-called drive equations providing thus the command signals for actuator control. Only very few authors try to handle the control problem on the basis of the dynamic equations for arm and actuators [1]. Therefore , in the following a typical articulated robotic arm with three degrees of freedom is described in detail. Its joints are driven by DC-motors, acting via gearings of relatively high ratios.

The highly nonlinear and strongly coupled dynamic model described in detail in the following may serve e.g. for testing

- classical and modern control algorithms based on the complete nonlinear model
- classical and modern control algorithms based on the linearized model with or without order reduction
- decoupling algorithms
- the effects of linearization, order reduction, etc.

2. The Robot.

The robot sketched in Fig.1 can be modelled as given by equs.(1) and (2). Control inputs are the control voltages U_{C_i} of the actuators. The controlled outputs are the joint angles φ_i and the relative joint angular velocities $\dot{\varphi}_i$ (i=1,2,3)

2.1 Drive equations

Notations:

Motor i mounted in link i-1 (link 0 = foundation) is driving φ_i

M_{A_i} ... electromagnetic moment acting on rotor R_i of the motor

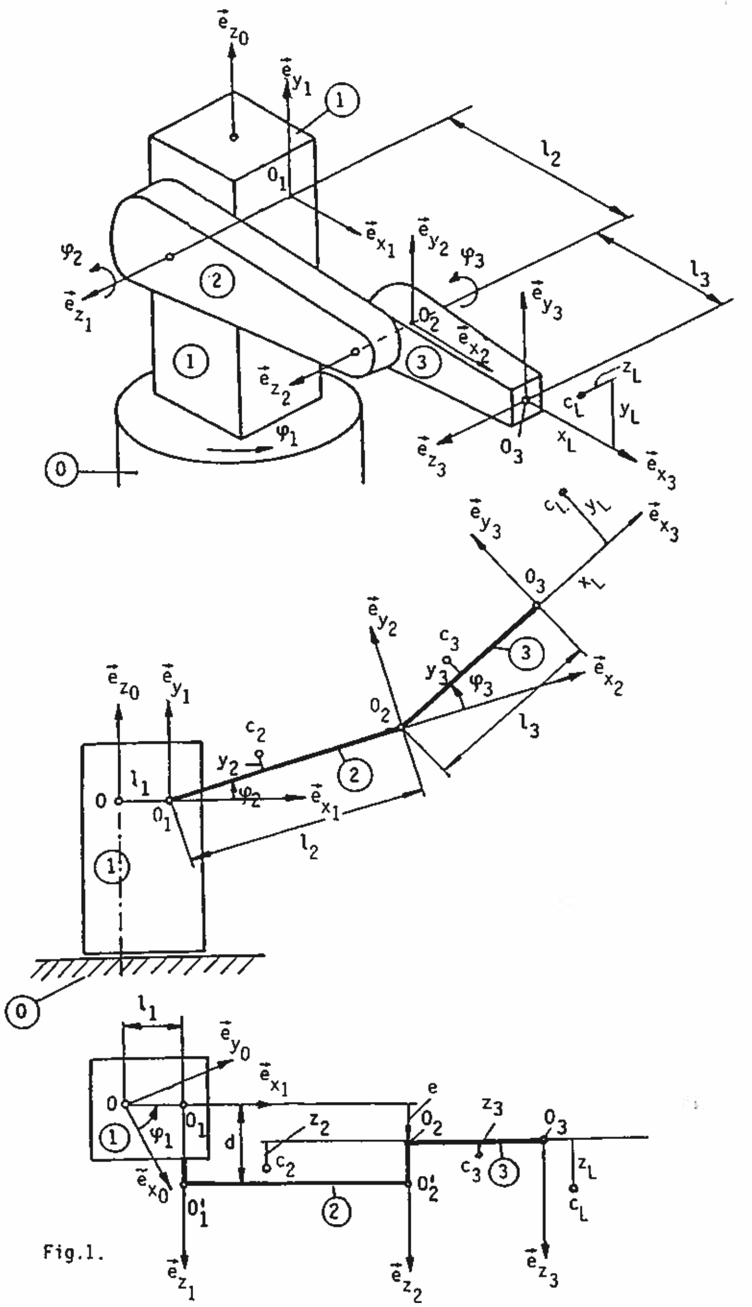

Fig.1.

[1]) Desoyer,K.; Kopacek,P.; Troch,I.: Industrieroboter und Handhabungsgeräte, R.Oldenbourg-Verlag, München, 1985.

M_{F_i} ... moment of friction reduced to the axis of rotor R_i (compare the assumption stated in 3.1)

ν_i ... gear ratio

σ_i ... relative angular velocity of rotor R_i in link i-1

$\sigma_i = \nu_i \dot{\varphi}_i \qquad (i=1,2,3)$

m_1, m_2, m_3 ... masses of the links 1,2,3 including actuators
m_L .... mass of end effector and load
c_i, c_L ...... center of mass = center of gravity of m_i, m_L
$\vec{e}_{x_i}, \vec{e}_{y_i}, \vec{e}_{z_i}$ .. unit vectors fixed in joint O_i of link i
x_i, y_i, z_i ... coordinates of c_i in the linkfixed system of the link i
x_L, y_L, z_L .... coordinates of c_L in the linkfixed system of link 3 (fig.1)

$C_i = \cos\varphi_i \qquad S_i = \sin\varphi_i \qquad C_{ik} = \cos(\varphi_i + \varphi_k) \qquad S_{ik} = \sin(\varphi_i + \varphi_k)$

$$m_i \begin{pmatrix} K_{x_i} & -K_{x_i y_i} & -K_{x_i z_i} \\ -K_{y_i x_i} & K_{y_i} & -K_{y_i z_i} \\ -K_{z_i x_i} & -K_{z_i y_i} & K_{z_i} \end{pmatrix}$$

(i=2,3) symmetric inertial tensor of link i with therein mounted actuator i+1, referred to the axes through origin O_{i-1} that are parallel to the linkfixed axes x_i, y_i, z_i.

$$m_L \begin{pmatrix} K_{x_L} & -K_{x_L y_L} & -K_{y_L z_L} \\ -K_{y_L z_L} & K_{y_L} & -K_{y_L z_L} \\ -K_{z_L x_L} & -K_{z_L y_L} & K_{z_L} \end{pmatrix}$$

symmetric inertial tensor of endeffector with load referred to the axes through origin O_2 that are parallel to the link fixed axes x_3, y_3, z_3 (fig.1).

I_1 ... inertial moment of link 1 with actuator 2 with respect to the spacefixed axis z_0

I_{R_i}... inertial moment of the rotor R_i of actuator i with respect to its axis $\vec{e}_i$ fixed in link i-1

$\alpha_i, \beta_i, \gamma_i$ ... angles between rotor axis $\vec{e}_i$ and linkfixed directions $\vec{e}_{x_{i-1}}, \vec{e}_{y_{i-1}}, \vec{e}_{z_{i-1}}$ (fig.2).

For simplification we assume that there is no relative motion between end-effector with load and link 3, (x_L, y_L, z_L = constant).

With the notations given above the following equations [2]) describe the electromagnetic moments M_{A_i} needed to act on the rotors of the actuators for a desired motion $\varphi_i(t)$ of the links (i=1,2,3)

$$\begin{pmatrix} \nu_1(M_{A_1} - M_{F_1}) \\ \nu_2(M_{A_2} - M_{F_2}) \\ \nu_3(M_{A_3} - M_{F_3}) \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \begin{pmatrix} \ddot{\varphi}_1 \\ \ddot{\varphi}_2 \\ \ddot{\varphi}_3 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} + \begin{pmatrix} M_{g1} \\ M_{g2} \\ M_{g3} \end{pmatrix} \qquad (1)$$

where the following abbreviations are used:
The elements m_{ik} of the 3x3 matrix in (1) are given by

$$m_{11} = I_1 + m_2\{K_{x_2}S_2^2 + K_{y_2}C_2^2 - 2K_{x_2y_2}S_2C_2 + 2[(l_2+x_2)C_2 - y_2S_2]l_1 + l_1^2\} +$$
$$+ m_3\{K_{x_3}S_{23}^2 + K_{y_3}C_{23}^2 - 2K_{x_3y_3}S_{23}C_{23} + e(e+2z_3) + (l_1+l_2C_2)^2 +$$
$$+ 2(l_1+l_2C_2)[(l_3+x_3)C_{23} - y_3S_{23}]\}$$
$$+ m_L\{K_{x_L}S_{23}^2 + K_{y_L}C_{23}^2 - 2K_{x_Ly_L}S_{23}C_{23} + e(e+2z_L) + (l_1+l_2C_2)^2 +$$
$$+ 2(l_1+l_2C_2)[(l_3+x_L)C_{23} - y_LS_{23}]\} + I_{R_1}\nu_1^2$$

$$m_{22} = m_2K_{z_2} + m_3\{K_{z_3} + l_2^2 + 2[(l_3+x_3)C_3 - y_3S_3]l_2\} +$$
$$+ m_L\{K_{z_L} + l_2^2 + 2[(l_3+x_L)C_3 - y_LS_3]l_2\} + I_{R_2}\nu_2^2$$

$$m_{33} = m_3K_{z_3} + m_LK_{z_L} + I_{R_3}\nu_3^2$$

$$m_{12} = m_{21} = -m_2(K_{z_2x_2}S_2 + K_{y_2z_2}C_2)$$
$$- m_3\{(e+z_3)l_2S_2 + (K_{y_3z_3} + ey_3)C_{23} + [K_{z_3x_3} + e(l_3+x_3)]S_{23}\}$$
$$- m_L\{(e+z_L)l_2S_2 + (K_{y_Lz_L} + ey_L)C_{23} + [K_{z_Lx_L} + e(l_3+x_L)]S_{23}\}$$
$$+ I_{R_2}\nu_2\cos\beta_2$$

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[2]) See footnote [1])

$$m_{13} = m_{31} = - m_3\{(K_{y_3z_3}+ey_3)C_{23} + [K_{z_3x_3} + e(l_3+x_3)]S_{23}\}$$

$$- m_L\{(K_{y_Lz_L}+ey_L)C_{23} + [K_{z_Lx_L} + e(l_3+x_L)]S_{23}\}$$

$$+ I_{R_3}\nu_3(S_2\cos\alpha_3+C_2\cos\beta_3)$$

$$m_{23} = m_{32} = m_3\{K_{z_3}+[(l_3+x_3)C_3-y_3S_3]l_2\} +$$

$$+ m_L\{K_{z_L}+[(l_3+x_L)C_3-y_LS_3]l_2\} + I_{R_3}\nu_3 \cos\gamma_3$$

The elements of the 3x1 matrix $\underline{n}$ in (1) are

$$n_1 = n_{1,22}\dot\varphi_2^2 + n_{1,33}\dot\varphi_3^2 + n_{1,12}\cdot 2\dot\varphi_1\dot\varphi_2 + n_{1,23}\cdot 2\dot\varphi_2\dot\varphi_3 + n_{1,31}\cdot 2\dot\varphi_3\dot\varphi_1$$

$$n_2 = n_{2,11}\dot\varphi_1^2 + n_{2,33}\dot\varphi_3^2 + n_{2,23}\cdot 2\dot\varphi_2\dot\varphi_3 + n_{2,31}\cdot 2\dot\varphi_3\dot\varphi_1$$

$$n_3 = n_{3,11}\dot\varphi_1^2 + n_{3,22}\dot\varphi_2^2 + n_{3,12}\cdot 2\dot\varphi_1\dot\varphi_2$$

with the abbreviations:

$$n_{1,22} = m_2(K_{y_2z_2}S_2 - K_{z_2x_2}C_2) +$$

$$+ m_3\{(K_{y_3z_3}+ey_3)S_{23}-[K_{z_3x_3}+e(l_3+x_3)]C_{23}-(e+z_3)l_2C_2\} +$$

$$+ m_L\{(K_{y_Lz_L}+ey_L)S_{23}-[K_{z_Lx_L}+e(l_3+x_L)]C_{23}-(e+z_L)l_2C_2\}$$

$$n_{1,33} = m_3\{(K_{y_3z_3}+ey_3)S_{23}-[K_{z_3x_3}+e(l_3+x_3)]C_{23}\} +$$

$$+ m_L\{(K_{y_Lz_L}+ey_L)S_{23}-[K_{z_Lx_L}+e(l_3+x_L)]C_{23}\}$$

$$n_{1,12} = m_2\{(K_{x_2}-K_{y_2})S_2C_2+K_{x_2y_2}(S_2^2-C_2^2)-[(l_2+x_2)S_2+y_2C_2]l_1\} +$$

$$+ m_3\{(K_{x_3}-K_{y_3})S_{23}C_{23}+K_{x_3y_3}(S_{23}^2-C_{23}^2)-(l_1+l_2C_2)[(l_3+x_3)S_{23}+y_3C_{23}]$$

$$- [(l_1+l_2C_2)+(l_3+x_3)C_{23}-y_3S_{23}]l_2S_2\} +$$

$$+ m_L\{(K_{x_L}-K_{y_L})S_{23}C_{23}+K_{x_Ly_L}(S_{23}^2-C_{23}^2)-(l_1+l_2C_2)[(l_3+x_L)S_{23}+y_LC_{23}\}$$

$$n_{1,23} = m_3\{(K_{y_3z_3}+ey_3)S_{23}-[K_{z_3x_3}+e(l_3+x_3)]C_{23}\}+$$

$$+ m_L\{(K_{y_Lz_L}+ey_L)S_{23}-[K_{z_Lx_L}+e(l_3+x_L)]C_{23}\}+$$

$$+ \frac{1}{2} I_{R_3}\nu_3(C_2\cos\alpha_3-S_2\cos\beta_3)$$

$$n_{1,31} = m_3\{(K_{x_3}-K_{y_3})S_{23}C_{23}+K_{x_3y_3}(S_{23}^2-C_{23}^2)-(l_1+l_2C_2)[(l_3+x_3)S_{23}+y_3C_{23}]\} +$$

$$+ m_L\{(K_{x_L}-K_{y_L})S_{23}C_{23}+K_{x_Ly_L}(S_{23}^2-C_{23}^2)-(l_1+l_2C_2)[(l_3+x_L)S_{23}+y_LC_{23}]\}$$

$n_{2,11} = - n_{1,12}$ $\quad n_{2,23} = n_{2,33}$ $\quad n_{2,31} = - \frac{1}{2} I_{R_3}\nu_3(C_2\cos\alpha_3-S_2\cos\beta_3)$

$$n_{2,33} = - m_3[(l_3+x_3)S_3+y_3C_3]l_2-m_L[(l_3+x_L)S_3+y_LC_3]l_2$$

$n_{3,11} = - n_{1,31}$ $\quad n_{3,22} = - n_{2,33}$ $\quad n_{3,12} = - n_{2,31}$

The moments M_{gi} in (1) due to the weight of the links and the load are:

$$M_{g1} = 0$$

$$M_{g2} = [(l_2+x_2)C_2-y_2S_2]m_2g + [(l_3+x_3)C_{23}-y_3S_{23}+l_2C_2]m_3g + [(l_3+x_L)C_{23}-y_LS_{23}+l_2C_2]m_Lg$$

$$M_{g3} = [(l_3+x_3)C_{23}-y_3S_{23}]m_3g + [(l_3+x_L)C_{23}-y_LS_{23}]m_Lg$$

2.2 Equations for the D.C.-motors:

With the notation

M_{Ai}	... electromagnetic drivemoment	(N.m)
K_i	... motorconstant	$(\frac{N.m}{A}) = (V.s)$
I_{Ai}	... armature current	(A)
L_{Ai}	... armature inductivity	$(\frac{V.s}{A})$
R_{Ai}	... armature Ohm resistance	(Ohm)
U_{Ai}	... armature voltage	(V)
U_{C_i}	... control voltage	(V)
$T_{St,i}$	.. time constant of the motor	(s)
$K_{St,i}$	.. gain	

the equations for the D.C.-motors are:

$$M_{Ai} = K_i I_{Ai}$$

$$L_{Ai}\frac{dI_{Ai}}{dt} + R_{Ai}I_{Ai} = U_{Ai} - K_i\sigma_i \qquad \text{with} \quad \sigma_i = \nu_i\dot{\varphi}_i \tag{2}$$

$$T_{St,i}\frac{dU_{Ai}}{dt} + U_{Ai} = K_{St,i}U_{C_i} \qquad (i=1,2,3)$$

2.3 Data

2.3.1 Data for the arm

$l_1 = 0{,}35$ m

$l_2 = 0{,}45$ m	$m_2 = 100$ kg	$x_2 = -0{,}60$ m	$y_2 = 0$	$z_2 = 0{,}20$ m
$l_3 = 0{,}20$ m	$m_3 = 25$ kg	$x_3 = -0{,}30$ m	$y_3 = 0$	$z_3 = 0$
$e = 0$	$m_L = 40$ kg	$x_L = 0{,}20$ m	$y_L = 0$	$z_L = 0$

$I_1 = 8{,}0$ kg.m^2

$K_{x_2} = 0{,}0436$ m^2	$K_{x_3} = 0{,}00130$ m^2	$K_{x_L} = 0{,}00330$ m^2
$K_{y_2} = 0{,}0681$ m^2	$K_{y_3} = 0{,}0223$ m^2	$K_{y_L} = 0{,}196$ m^2
$K_{z_2} = 0{,}0625$ m^2	$K_{z_3} = 0{,}0243$ m^2	$K_{z_L} = 0{,}192$ m^2
$K_{x_2y_2} = K_{y_2x_2} = 0$	$K_{x_3y_3} = K_{y_3x_3} = 0$	$K_{x_Ly_L} = K_{y_Lx_L} = 0$
$K_{y_2z_2} = K_{z_2y_2} = 0$	$K_{y_3z_3} = K_{z_3y_3} = 0$	$K_{y_Lz_L} = K_{z_Ly_L} = 0$
$K_{z_2x_2} = K_{x_2z_2} = -0{,}0300$ m^2	$K_{z_3x_3} = K_{x_3z_x} = 0$	$K_{z_Lx_L} = K_{x_Lz_L} = 0$

2.3.2 Data for the D.C.-motors:

for $i = 1,2,3$

	gear ratios:	moments of friction: [3]
$K_i = 0{,}167$ V.s		
$L_{A_i} = 8.10^{-5}$ V.s/A	$\nu_1 = 220$	$M_{F_1} = 0{,}245.\text{sign}\,\dot{\varphi}_1$ N.m
$R_{A_i} = 1{,}50$ Ohm	$\nu_2 = 150$	$M_{F_2} = 0{,}313.\text{sign}\,\dot{\varphi}_2$ N.m
$I_{R_i} = 0{,}000235$ kg.m^2	$\nu_3 = 130$	$M_{F_3} = 0{,}397.\text{sign}\,\dot{\varphi}_3$ N.m
$T_{St,i} = 0{,}012$ s		
$K_{St,i} = 1$		$\cos\alpha_3 = 1$
	$\cos\beta_2 = 0$	$\cos\beta_3 = 0$
		$\cos\gamma_3 = 0$

[3] Compare the assumption stated in 3.1

3.1 Control task:

Open-loop or closed-loop controls have to be designed for point-to-point movements and for continuous path movements.

The following important assumption is made: whenever a joint angle φ_i is constant over a time-interval of positive length (i.e. before starting the movement or after reaching its final value), a mechanical brake becomes active and fixes this joint angle. Therefore, from this moment on the i-th equation in (1) and (2) respectively, have to be cancelled.

3.1.1. Point-to-point movements.

Initial and final positions $\varphi_i(0)$ and $\varphi_i(t_f)$ are given for i=1,2,3, initial and final joint angular velocities $\dot{\varphi}_i$ have to equal zero ($\dot{\varphi}_i(0) = \dot{\varphi}_i(t_f) = 0$, i=1,2,3). The final time t_f may be given or open (time-optimal control).
Further, bounds

$$|\dot{\varphi}_i| \le \dot{\varphi}_i^* \qquad |I_{A_i}| \le I_{A_i}^* \qquad |U_{A_i}| \le U_{A_i}^* \qquad i = 1,2,3$$

must not be exceeded for $0 \le t \le t_f$.

In addition command angular velocities may be given as indicated in Fig.3 with $|\dot{\varphi}_{i,max}| \le \dot{\varphi}_i^*$

3.1.2. Continuous path control

A smooth path $\varphi_i(t)$, i=1,2,3, $0 \le t \le t_f$ is given. The control voltages U_{C_i} are to be designed such that the actual movement equals the desired one and such that the bounds hold on $0 \le t \le t_f$.

3.1.3. Data:

$\dot{\varphi}_i^* = \sigma_{i,max}/\nu_i$ with $\sigma_{i,max} = 314$ rad/s

$I_{A_i}^* = 8$ A

$U_{A_i}^* = 65$ V

Curve in Fig.3:

$$\dot{\varphi}_i = \begin{cases} 0 & t \le t_{i,1} \\ \frac{\dot{\varphi}_{i,max}}{2}\left[1-\cos\pi\left(\frac{t-t_{i,1}}{t_{i,2}-t_{i,1}}\right)\right] & t_{i,1} \le t \le t_{i,2} \\ \dot{\varphi}_{i,max} & t_{i,2} \le t \le t_{i,3} \\ \frac{\dot{\varphi}_{i,max}}{2}\left[1+\cos\pi\left(\frac{t-t_{i,3}}{t_{i,4}-t_{i,3}}\right)\right] & t_{i,3} \le t \le t_{i,4} \\ 0 & t_{i,4} \le t \le t_f \end{cases}$$

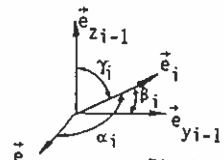

Fig.2.

For variation of mass and position of endeffector with load there are the relations:

$$K_{x_L} = k_{x_L} + y_L^2 + z_L^2 \qquad K_{x_L y_L} = k_{x_L y_L} + (l_3 + x_L) y_L$$

$$K_{y_L} = k_{y_L} + z_L^2 + (l_3 + x_L)^2 \qquad K_{y_L z_L} = k_{y_L z_L} + y_L z_L$$

$$K_{z_L} = k_{z_L} + (l_3 + x_L)^2 + y_L^2 \qquad K_{z_L x_L} = k_{z_L x_L} + z_L (l_3 + x_L)$$

wherein $m_L \begin{pmatrix} k_{x_L} & -k_{x_L y_L} & -k_{x_L z_L} \\ -k_{y_L x_L} & k_{y_L} & -k_{y_L z_L} \\ -k_{z_L x_L} & -k_{z_L y_L} & k_{z_L} \end{pmatrix}$ symmetric inertial tensor of endeffector with load with respect to the axes through the center of mass c_L that are parallel to the axes x_3, y_3, z_3 of link 3.

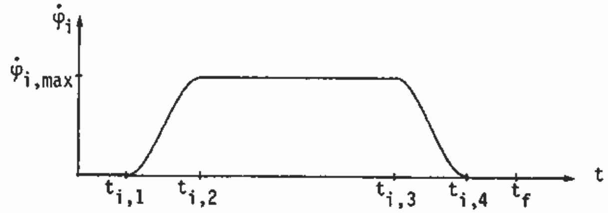

Fig.3.

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General Systems

Problem #90-13 (General Control of a 'Simple' Process)

Problem #90-13 ROBUST CONTROL OF A 'SIMPLE' PROCESS

(A) General Description

This problem describes a relatively 'simple' problem which can be used for evaluating a general purpose adaptive controller (such as those adaptive controllers which are now commercialized by ASEA (Novatune), Foxboro (Exact), Turnbull (Model 6355), etc.) or any other fixed gain robust controller.

(B) Reference

The formulation of this problem is given in:

Larminat Ph. de., "Adaptive PID Regulators: Ambitions and Limitations", *1988 IFAC Workshop on Robust Adaptive Control*, Newcastle, Australia, Aug. 1988.

(C) Problem Description

The system to be controlled is a SISO system described by the following block diagram (see Figure 1):

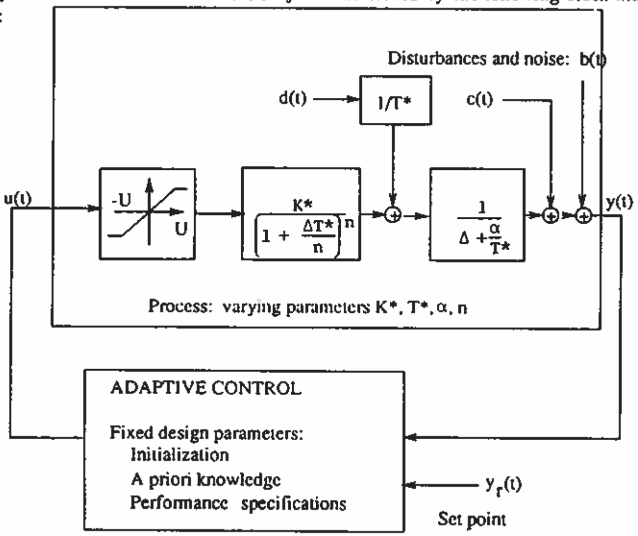

Figure 1: Adaptive control of a simple process

where the following notation is used:

Notation

t	=	$0,1,2,3,\ldots$
Δ	:	finite difference operator
$\Delta x(t)$	=	$x(t+1)-x(t)$
$u(t)$	:	control input
$y(t)$	:	output
$y_r(t)$	:	set point
$d(t), c(t)$	:	load disturbances
$b(t)$	:	measurement noise
U	:	level of input saturation = 6

It is desired to find a *fixed* self-tuning regulator (or any other type of adaptive controller or fixed gain robust controller) ***to control the largest possible family of plants*** with the structure as given in Figure 1 such that the closed loop system is stable and "good" tracking/regulation occurs in the system. In particular, it is assumed that the parameters K^*, T^*, n, α of the plant are uncertain (and possibly are subject to change), and are given as follows:

$$10 \le T^* \le 100$$
$$0.1 \le K^* \le 10$$
$$-0.1 \le \alpha \le 0.2$$
$$2 \le n \le 8$$

Discussion

The problem above is a discrete system representation, and can be interpreted when $n \to \infty$, $\alpha \to 0$, $T^* \gg 1$, as being an approximation to a plant with the following transfer function:

$$y(s) = K^* \frac{e^{-sT^*}}{s} u(s)$$

Class of Disturbances/Set Points

(a) It is assumed that $b(t)$ is a coloured noise given by:

$$b(t) = \left(\frac{1}{1+\Delta\tau}\right) n(t), \quad \text{where} \quad \tau \le 0.25T^*$$

where $n(t)$ is a sequence of independent elements with zero mean and with variance:

$$V = \sigma_b^2(2\tau - 1), \quad \text{where} \quad 0 \le \sigma_b \le 0.2$$

which implies that the variance of $b(t)$ is given by σ_b^2.

(b) It is assumed that $y_r(t)$ arises from a class of square waves with amplitude varying from 0 to 10 and with period equal to $20T^*$.

(c) The load disturbances $c(t)$ or/and $d(t)$ are assumed to be square waves with amplitude varying from 0 to 5 and with period equal to $20T^*$.

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