IFAC Professional Brief

Sensor Fusion

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Abstract

Sensor fusion is a method of integrating signals from multiple sources. It allows extracting information from several different sources to integrate them into single signal or information. In many cases sources of information are sensors or other devices that allow for perception or measurement of changing environment. Information received from multiple-sensors is processed using "sensor fusion" or "data fusion" algorithms. These algorithms can be classified into three different groups. First, fusion based on probabilistic models, second, fusion based on least-squares techniques and third, intelligent fusion. The probabilistic model methods are Bayesian reasoning, evidence theory, robust statistics, recursive operators. The least-squares techniques are Kalman filtering, optimal theory, regularization and uncertainty ellipsoids. The intelligent fusion methods are fuzzy logic, neural networks and genetic algorithms. This paper will present three different methods of intelligent information fusion for different engineering applications. Chapter 2 is based on Sasiadek and Wang (2001) paper and presents an application of adaptive Kalman filtering to the problem of information fusion for guidance, navigation, and control. Chapter 3 is based on Sasiadek and Hartana (2000) and Chapter 4 on Sasiadek and Khe (2001) paper.

1. Introduction

The data/sensor problems and related methods are used in conjunction to many engineering applications. For example, guidance, navigation, and control of vehicles require large number of information from different sources. This information often is similar and has to be integrated into one meaningful signal or

information that can be used in control systems. In this paper the sensor/data fusion will be shown for three different cases. In all three cases the Kalman Filter method is used to integrate signals/information received from multiple-sensor sources. Also, in all three cases the modification of Kalman Filter method is introduced to improve performance. This modification is based on Fuzzy Logic System (FLS). In that sense, the integration method becomes an intelligent integration, and is applicable to broader number of industrial cases. The chapter 2 is presenting an integration of data/sensor signals received from the Global Positioning System (GPS) and Inertial Navigation System (INS). The integration allows for better and more accurate positioning.
Chapter 3 presents the navigation of an autonomous robot based on sensor/data fusion method for signals received from sonar and odometry sensors. The fusion process allows for more efficient navigation and obstacle avoidance. In both cases described in chapter 2 and 3 the Kalman Filter method is backed up by the Fuzzy Logic System (FLS).
Finally, chapter 4 is presenting an attempt to design integration method based fully on FLS. Results and conclusions are shown separately for those three different cases.

2. Fuzzy Adaptive Kalman Filtering for INS/GPS Data Fusion and Accurate Positioning

2.1 Introduction

This chapter presents the method of sensor fusion based on the Adaptive Fuzzy Kalman Filtering. This method has been applied to fuse position signals from the Global Positioning System (GPS) and Inertial Navigation System (INS) for the autonomous mobile vehicles. The presented method has been validated in 3-D environment and is of particular importance for guidance, navigation, and control of flying vehicles. The Extended Kalman Filter (EKF) and the noise characteristic have been modified using the Fuzzy Logic Adaptive System and compared with the performance of regular EKF. It has been demonstrated that the Fuzzy Adaptive Kalman Filter gives better results (more accurate) than the EKF

2.2 Sensor Fusion

When navigating and guiding an autonomous vehicle, the position and velocity of the vehicle must be determined. The Global Positioning System (GPS) is a satellite-based navigation system that provides a user with the proper equipment access to useful and accurate positioning information anywhere on the globe (see Brown and Hwang, 1992). However, several errors are associated with the GPS measurement. It has superior long-term error performance, but poor short-term accuracy. For many vehicle navigation systems, GPS is insufficient as a stand-alone position system. The integration of GPS and Inertial Navigation System (INS) is ideal for vehicle navigation systems. In generally, the short-term accuracy of INS is good; the long-term accuracy is poor. The disadvantages of GPS/INS are ideally cancelled. If the signal of GPS is interrupted, the INS enables the navigation system to coast along until GPS signal is reestablished. The requirements for accuracy, availability and robustness are therefore achieved.

Kalman filtering is a form of optimal estimation characterized by recursive evaluation, and an internal model of the dynamics of the system being estimated. The dynamic weighting of incoming evidence with ongoing expectation produces estimates of the state of the observed system (see Abidi and Gonzalez, 1992). An extended Kalman filter (EKF) can be used to fuse measurements from GPS and INS. In this EKF, the INS data are used as a reference trajectory, and GPS data are applied to update and estimate the error states of this trajectory. The Kalman filter requires that all the plant dynamics and noise processes are exactly known and the noise processes are zero mean white noise. If the theoretical behavior of a filter and its actual behavior do not agree, divergence problems will occur. There are two kinds of divergence: Apparent divergence and True divergence (Gelb, 1992). In the apparent divergence, the actual estimate error covariance remains bounded, but it approaches a larger bound than does predicted error covariance. In true divergence, the actual estimation covariance eventually becomes infinite. The divergence due to modeling errors is critical in Kalman filter application. If, the Kalman filter is fed information that the process behaved one way, whereas, in fact, it behaves another way, the filter will try to continually fit a wrong process. When the measurement situation does not provide enough information to estimate all the state variables of the system, in other words, the computed estimation error matrix becomes unrealistically small, and the filter disregards the measurement, then the problem is particularly severe. Thus, in order to solve the divergence due to modeling errors, we can estimate unmodeled states, but it adds complexity to the filter and one can never be sure that all of the suspected unstable states are indeed model states (Lewis, 1986). Another possibility is to add process noise. It makes sure that the Kalman filter is driven by white noise, and prevents the filter from disregarding new measurement. In this paper, a fuzzy logic adaptive system (FLAS) is used to adjust the exponential weighting of a weighted EKF and prevent the Kalman filter from divergence. The fuzzy logic adaptive controller (FLAC) will continually adjust the noise strengths in the filter's internal model, and tune the filter as well as possible. The FLAC performance is evaluated by simulation of the fuzzy adaptive extended Kalman filtering scheme of Fig.1.

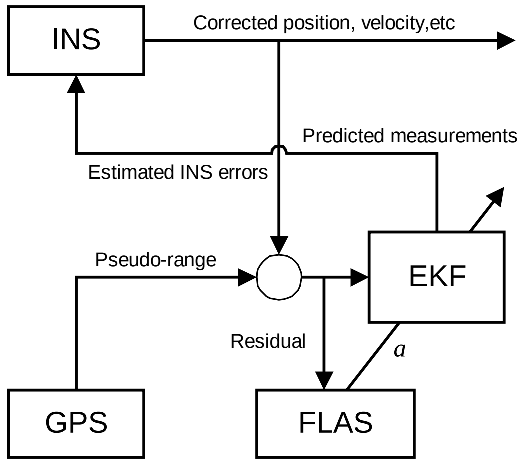


Fig.1. Fuzzy adaptive extended Kalman filter

2.2.1 Weighted EKF

Because the processes of both GPS and INS are nonlinear, a linearization is necessary. An extended Kalman filter is used to fuse the measurements from the GPS and INS. To prevent divergence by keeping the filter from discounting measurements for large *k*, the exponential data weighting (Lewis, 1986) is used.

The models and implementation equations for the weighted extended Kalman filter are:

Nonlinear dynamic model

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, k) + \mathbf{w}_k \qquad (1)$$
$$\mathbf{w}_k \sim N(0, \mathbf{Q})$$

Nonlinear measurement model

$$\mathbf{z}_k = h(\mathbf{x}_k, k) + \mathbf{v}_k \qquad (2)$$
$$\mathbf{v}_k \sim N(0, \mathbf{R})$$

Let us set the model covariance matrices equal to

$$\mathbf{R}_k = \mathbf{R}a^{-2(k+1)} \qquad (3)$$
$$\mathbf{Q}_k = \mathbf{Q}a^{-2(k+1)} \qquad (4)$$

where, α≥1, and constant matrices **Q** and **R**. For α>1, as time k increases, the **R** and **Q** decrease, so that the most recent measurement is given higher weighting. If α=1, it is a regular EKF.

By defining the weighted covariance

$$\mathbf{P}_k^{a-} = \mathbf{P}_k^{-} a^{2k} \tag{5}$$

The Kalman gain can be computed:

$$\begin{aligned} \mathbf{K}_k &= \mathbf{P}_k^{-}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_k^{-}\mathbf{H}_k^T + \mathbf{R}a^{-2(k+1)})^{-1} \\ &= \mathbf{P}_k^{a-}\mathbf{H}_k^T\left(\mathbf{H}_k\mathbf{P}_k^{a-}\mathbf{H}_k^T + \frac{\mathbf{R}}{a^2}\right)^{-1} \end{aligned} \tag{6}$$

The predicted state estimate is:

$$\hat{\mathbf{x}}_{k+1}^{-} = f(\hat{\mathbf{x}}_k, k) \tag{7}$$

The predicted measurement is:

$$\hat{\mathbf{z}}_k = h(\hat{\mathbf{x}}_k^{-}, k) \tag{8}$$

The linear approximation equations can be presented in form:

$$\Phi_k \approx \left.\frac{\partial f(x,k)}{\partial x}\right|_{x=\hat{x}_k^-} \tag{9}$$

The predicted estimate on the measurement can be computed:

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^{-} + \mathbf{K}_k(\mathbf{z}_k - \hat{\mathbf{z}}_k) \tag{10}$$

$$\mathbf{H}_k \approx \left.\frac{\partial h(x,k)}{\partial x}\right|_{x=x_k^-} \tag{11}$$

Computing the *a priori* covariance matrix:

$$\mathbf{P}_{k+1}^{-} = \Phi_k\mathbf{P}_k\Phi_k^T + \mathbf{Q}a^{-2(k+1)} \tag{12}$$

Re-writing (12) gives:

$$\mathbf{P}_{k+1}^{a-} = a^2\Phi_k\mathbf{P}_k^a\Phi^T + \mathbf{Q} \tag{13}$$

Computing the *a posteriori* covariance matrix gives:

$$\mathbf{P}_k^a = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)\mathbf{P}_k^{a-} \quad (14)$$

The initial condition is:

$\mathbf{P}_0^{a-} = \mathbf{P}_0$

In equation (10), the term $\mathbf{z}_k - \hat{\mathbf{z}}_k$ is called residuals or innovations. It reflects the degree to which the model fits the data.

2.2.2 INS and GPS

The inertial navigation system (INS) consists of a sensor package, which includes accelerometers and gyros to measure accelerations and angular rates. By using these signals as input, the attitude angle and three-dimensional vectors of velocity and position are computed (Jochen et al., 1994). The errors in the measurements of force made by the accelerometers and the errors in the measurement of angular change in orientation with respect to inertial space made by gyroscopes are two fundamental error sources, which affect the error behavior of an inertial system. The inertial system error response, related to position, velocity, and orientation is divergent with time due to noise input (Kayton and Fried, 1997). There are biases associated with the accelerometers and gyros. In order to correct the errors of INS, the GPS measurements are used to estimate the inertial system errors, subtract them from the INS outputs, and then obtain the corrected INS outputs. There is number of errors in GPS, such as ephemeris errors, propagation errors, selective availability, multi-path, and receiver noise, etc. Using differential GPS (DGPS), most of the errors can be corrected, but the multi-path and receiver noise cannot be eliminated.

2.3 Fuzzy Logic Adaptive System

It is assumed that both, the process noise $\mathbf{w}_{k,}$ and the measurement noise $\mathbf{v}_k$ are zero-mean white sequences with known covariance **Q** and **R** in the Kalman filter. If the Kalman filter is based on a complete and perfectly tuned model, the residuals or innovations should be a zero-mean white noise process. If the residuals are not white noise, there is something wrong with the design and the filter is not performing optimally (Lewis, 1986). The Kalman filters will diverge or coverage to a large bound. In practice, it is difficult to know the exact values for **Q** and **R.** In order to reduce computation, we have to ignore some errors, but sometimes those unmodeled errors will become significant. These are the instrument bias errors of INS. Sometimes the $\mathbf{w}_k$ may be different than zero mean. In those cases, the residuals can be used to adapt the filter. In fact, the residuals are the differences between actual measurements and best measurement predictions based on the filter's internal model. A well-tuned filter is that where the 95% of the autocorrelation function of innovation series should fall within the $\pm\ 2\sigma$ boundary (Cooper and Dyrrant-White, 1994). If the filter diverges, the residuals will not be zero mean and become larger.

There are very few papers on application of fuzzy logic to adapt the Kalman filter. Other authors (Abdelnour et al., 1993)), use fuzzy logic for on-line detection, and correction of divergence in a single state Kalman filter. There were three inputs and two outputs to fuzzy logic controller (FLC), and 24 rules were used. In our works (Sasiadek and Wang, 1999), the basic adaptive fuzzy logic controller has been introduced and designed. In this paper the new FLAC is proposed. The purpose of the fuzzy logic adaptive controller (FLAC) is to detect the bias of measurements and prevent divergence of the extended Kalman filter. It has been applied in three axes — East (x), North (y), and Altitude (z). The covariance of the residuals and mean values of residuals are used to decide the degree of divergence. The value of covariance relates to **R**. If the residual has zero mean, the equation for covariance of the residual is:

$$\overline{\mathbf{P}}_z = \mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R} \qquad (15)$$

The fuzzy adaptive Kalman filtering has been used for guidance and navigation of mobile robots, especially for 3-D environment. The navigation of flying robots requires fast, and accurate on-line control algorithms. The "regular" Extended Kalman Filter requires high number of states for accurate navigation and positioning and is unable to monitor the parameters changing. The FLAC requires smaller number of states for the same accuracy and therefore it would need less computational effort. Alternatively, the same number of states (as in "regular" filter) would allow for more accurate navigation.

2.3.1 Fuzzy adaptive Kalman filtering for parameter uncertainties

Sometimes, uncertain or time varying parameters are considered to exist in the **Q** and **R** matrices. The fuzzy adaptive Kalman filtering is used to detect and then adapt the filter on-line. There are two groups of fuzzy controllers. In those two fuzzy controllers, the covariance of the residuals and the mean of residuals are used as the inputs to both controllers for all three fuzzy inference engines. The exponential weighting a for first group controller and the scales for second group controller of three axes are the outputs.

The first group, which output is a, was used to detect the filter divergence and adapt the EKF. Generally, when the covariance is becoming large, and mean value is moving away from zero, the Kalman filter is becoming unstable. In this case, a large a will be applied. A large a means that process noises are added. It can ensure that in the model all states are sufficiently excited by the process noise. When the covariance is extremely large, there are some problems with the GPS measurements, so the filter cannot depend on these measurements anymore, and a smaller a will be used. By selecting an appropriate, a, the fuzzy logic controller will adapt the Kalman filter optimally and try to keep the innovation sequence acting as zero-mean white noise. Some membership functions are shown at figure 2, 3 and 4.

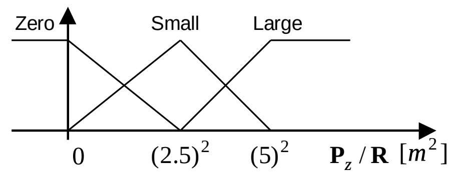


Fig.2. Covariance Membership Functions

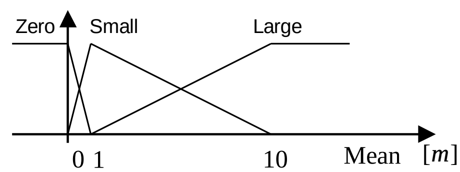


Fig.3. Mean Value Membership Functions

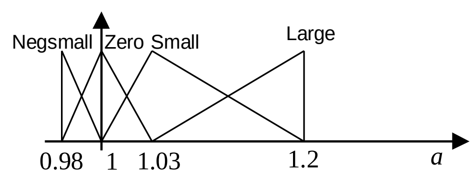


Fig.4. a Membership Functions

The fuzzy logic controller uses 9 rules, such as:
If *the covariance of residuals is large* ***and*** *the mean value is zero* ***Then*** *a is zero.*
If *the covariance of residuals is zero* ***and*** *the mean value is large* ***Then*** *a is small.*

The second group, which output is scale, was used to detect the change of measurement noise covariance **R**. From equation (15), the **R** is related to the covariance of residual, the larger the covariance of residual, the more the measurement noise. When the fuzzy logic controller finds that the covariance of residual is larger than that expected, it applies a large scale to adjust the a. A sample rule is:

***If** the covariance of residuals is small **and** the mean values is small **then** the scale is large.*

Table 1 and 2 are the rule table for those two groups of fuzzy controllers.

Table. 1. Rule Table for a

a		Mean Value		
		Z	**S**	**L**
P	Z	**Z**	**S**	**S**
	S	**S**	**L**	**S**
	L	**Z**	**NS**	**NS**

S --- **S**mall; **L** --- **L**arge;
Z --- **Z**ero; **NS --- N**egative **S**mall

Table. 2 Rule Table for Scale

Scale		Mean Value		
		Z	**S**	**L**
P	Z	**Z**	**Z**	**Z**
	S	**S**	**S**	**S**
	L	**L**	**S**	**Z**

2.3.2 Fuzzy adaptive Kalman filtering for non-white process noise

It is assumed that the process noise $\mathbf{w}_k$ is white noise for Kalman filtering. But sometime the process noise could be correlated with itself, non-white. In this case, we can add a shaping filtering that manufactures colored noise $\mathbf{w}_k$ with a given spectral density from white noise, but it will increase the state variables. In some real-time situation, the computing time have a restriction for increasing the state variables. We can use a fuzzy adaptive Kalman filtering to adaptive the process noise rather than add more state variables. There are 9 rules and therefore, little computational time is needed. The membership functions for this fuzzy control are showed as figure 5.8, 5.9, and 5.10.

The FLAC uses 9 rules, such as:

If *the covariance of residuals is large* ***and*** *the mean values are zero* ***Then*** *a is large.*
If *the covariance of residuals is zero* ***and*** *the mean values are large* ***Then*** *a is zero.*

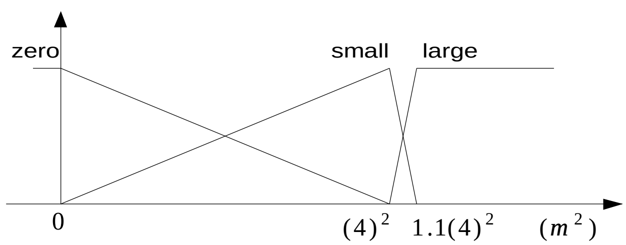


Fig.5. Covariance Membership Functions

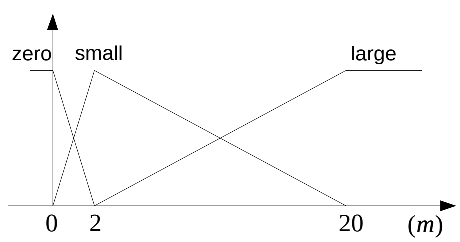


Fig.6. Mean Value Membership Functions

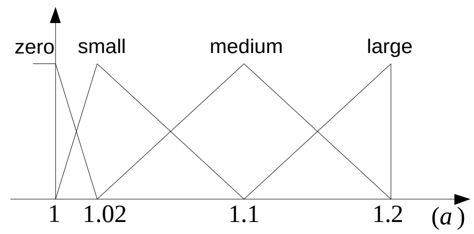


Fig.7. a Membership Functions

Table. 3. Rule Table for FLAS

a		Mean Value		
		Z	**S**	**L**
P	Z	**S**	**Z**	**Z**
	S	**Z**	**L**	**M**
	L	**L**	**M**	**Z**

S --- **S**mall; **M** --- **M**edium;
L --- **L**arge; **Z** --- **Z**ero;

2.4 Simulation

MATLAB codes developed by authors has been used to simulate and test the proposed method.

The state variables used in simulation are:

$$\mathbf{x}_k = [x_k, \dot{x}_k, y_k, \dot{y}_k, z_k, \dot{z}_k, c\Delta t, c\dot{\Delta t}] \qquad (16)$$

The states are position, and velocity errors of the INS East, North, Altitude, GPS range bias and range drift.

2.4.1 Simulation experiment 1

The first part of simulation uses the fuzzy adaptive Kalman filtering for parameter uncertainties.

The designed standard deviation of GPS measurement **R** is 5 [m]. The designed standard deviations of **Q** for INS are 0.0012 meter, 0.0012 meter, and 0.0012 meter for the East (x), North (y), and Altitude (z) respectively.

The simulations (Table 4, 5 and 6 and Figure 8 and 9) show that after the filter is stabilized, the actual error covariance of fuzzy logic adaptive EKF almost agrees with the theory error covariance. In the Table 4, 5 and 6, the designed parameters are **Q** and **R**. The 5**Q**, 2**R** etc. mean that the real time parameters are 5 and 2 time as large as the designed **Q** and **R**. In figure 8, and 9, dashed lines are the theoretical covariance of EKF, and the solid lines are the covariance of fuzzy adaptive EKF.

Table 4 Comparison of theoretical and actual error variance (X-axis)

Q	R	Theory	Actual
5Q	R	3.1711	3.3912
5Q	2R	5.3293	5.3121
3Q	2R	4.6896	4.8469
5Q	4R	8.9612	8.3122

Table 5 Comparison of theoretical and actual error variance (Y-axis)

Q	R	Theory	Actual
5Q	R	2.5540	2.7227
5Q	2R	4.2877	4.1030
3Q	2R	3.7694	4.0864
5Q	4R	7.2002	7.7340

Table 6 Comparison of theoretical and actual error variance (Z-axis)

Q	R	Theory	Actual
5Q	R	0.8344	0.8072
5Q	2R	1.3979	1.1796
3Q	2R	1.2268	1.2989
5Q	4R	2.3417	2.5005

2.4.2 Simulation experiment 2

In the second set of simulations, we simulate the inputs of non-white process noise. The covariance of GPS measurement R is 25 [m^2]. It is assumed that the measurements of INS have some biases. In the first part of this simulation (Fig. 5), the mean values of INS are 0.0014 meter, 0.00035 meter, and 0.0007 meter for the East (x), North (y), and Altitude (z) respectively. A white noise with a standard deviation of 3 meter is added to GPS measurements. The sample period is 1 second. The first row in Fig. 10 is the innovations of fuzzy adaptive EKF and EKF in East (x). The innovation of EKF had a large drift, and was stable at a high mean value. The fuzzy adaptive EKF clearly improved the performance of EKF, and the mean value was much smaller than that of EKF. Other figures present the corrected position (first column) and velocity (second column) errors. The corrected error is the current INS error minus estimated INS error. The dashed lines are the corrected errors of EKF, and the solid lines are the corrected errors of fuzzy adaptive EKF. The fuzzy adaptive EKF significantly reduced the corrected position and velocity errors. In the second part of this simulation (Fig. 11), the same measurements as in the first part of this simulation for INS were used. A white noise with a standard deviation of 2 meter from 0 s to 1000 s and 1500 s to 2000s was applied to GPS measurements. From 1000 s to 1500 s, the standard deviation of 6 meter with mean value of 6 meter was added to GPS measurements. Although, the GPS measurement noises features were changed, the fuzzy adaptive EKF still worked well. Those simulations also showed that the corrected errors of EKF were proportional to the mean values of INS measurements. In other word, the more errors are not modeled, the worse the EKF performs.

2.5 Summary

In this chapter, a fuzzy adaptive extended Kalman filter has been developed to detect and prevent the EKF from divergence. By monitoring the innovations sequences, the FLAS can evaluate the performance of an EKF. If the filter does not perform well, it would apply an appropriate weighting factor a to improve the accuracy of an EKF.

The FLAS can use lower order state-model without compromising accuracy significantly. Other words, for any given accuracy, the fuzzy adaptive Kalman filter may be able to use a lower order state model. The FLAS makes the necessary trade-off between accuracy and computational burden due to the increased dimension of the error state vector and associated matrices. When a designer lacks sufficient information to develop complete models or the parameters will slowly change with time, the fuzzy controller can be used to adjust the performance of EKF on-line, and it will remain sensitive to parameter variations by "remembering" most recent N data samples. It can be used to navigate and guide autonomous vehicles or robots and achieved a relatively accurate performance.

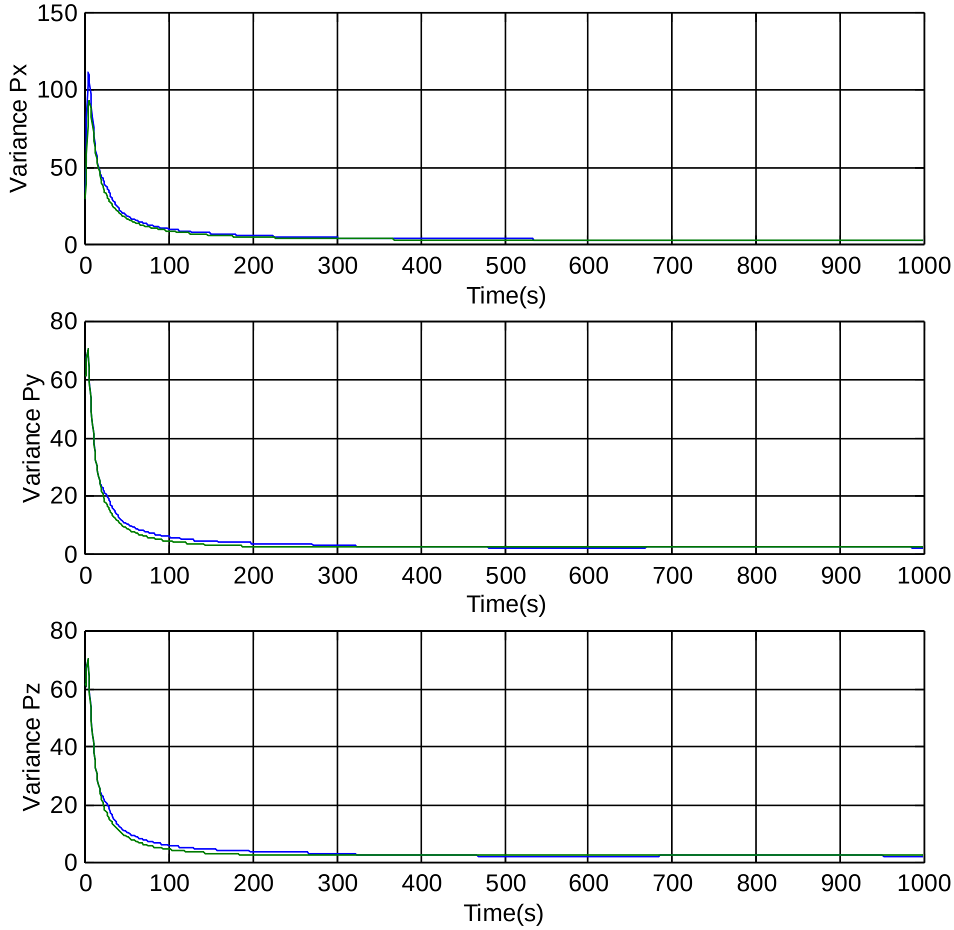


Fig. 8. Actual and Theory Covariance for 5Q and R

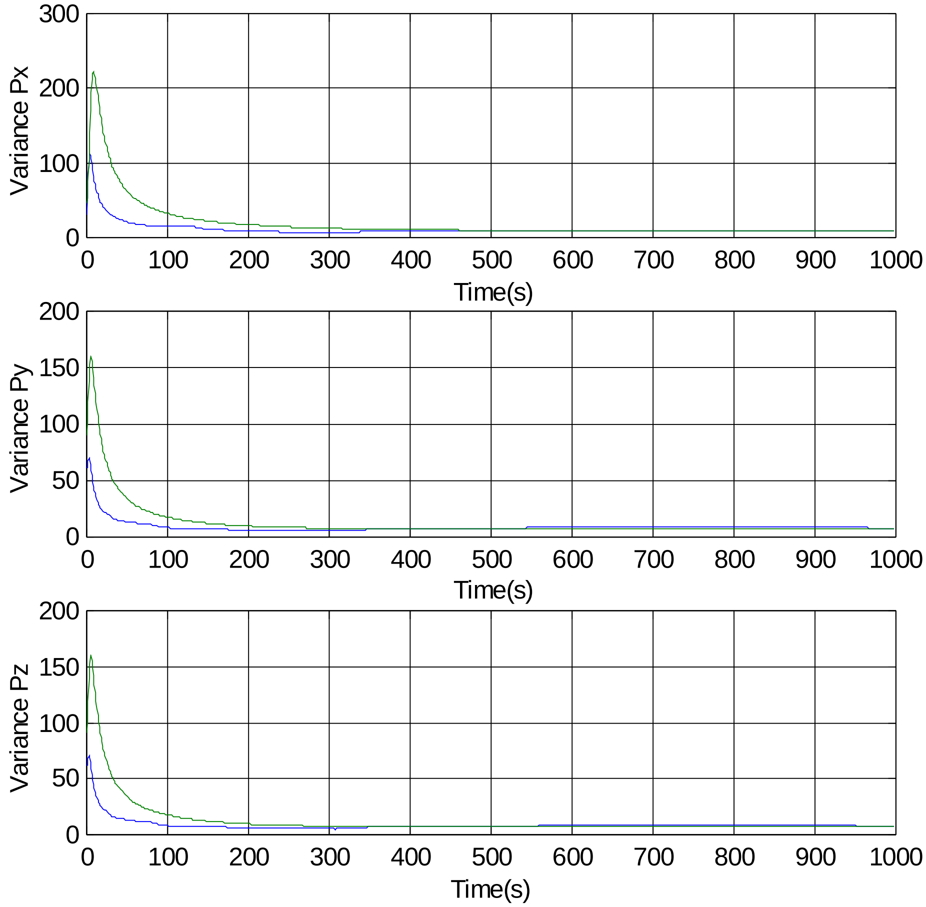


Fig. 9. Actual and Theory Covariance for 5Q and 4R

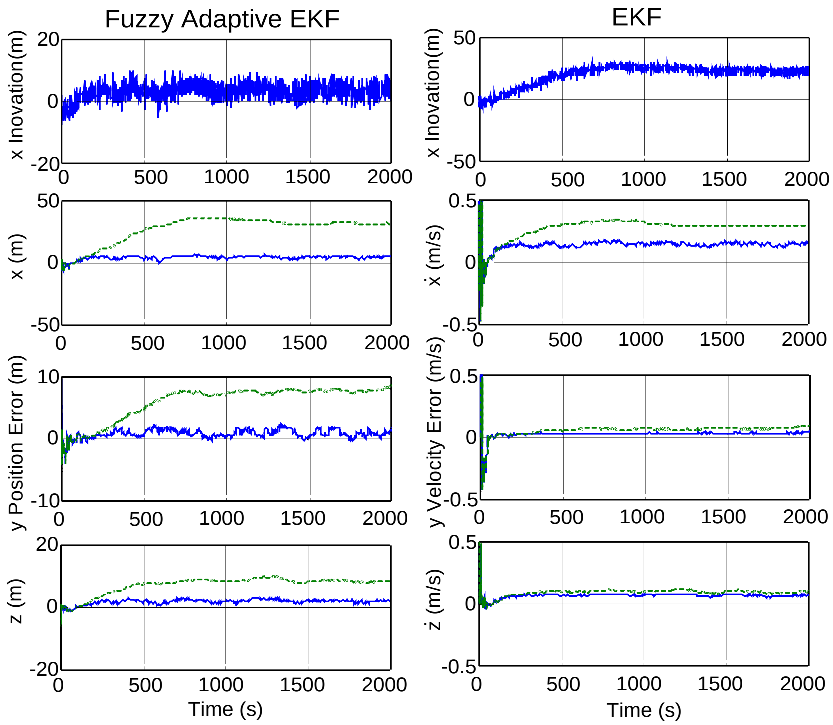


Fig. 10. Simulation A

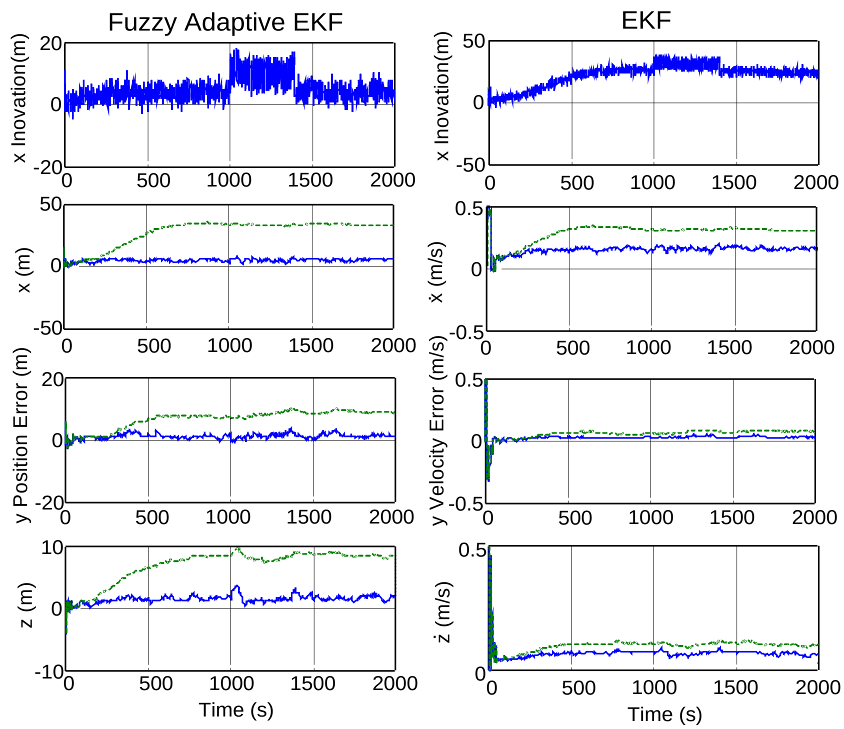


Figure 11. Simulation B

3. Sensor Fusion for dead-reckoning mobile robot navigation

3.1 Introduction

In positioning and localization problems, two or more different sensors are often used to obtain the best estimation data for control system. Extended Kalman Filter (EKF) is widely used to fuse those data to obtain one optimal result. One consideration when using EKF is the signal used during navigation is a white noise signals. This consideration is hardly found in real application. This paper presents the sensor fusion for dead-reckoning mobile robot navigation. Odometry and sonar signals are fused using Extended Kalman Filter (EKF) and Adaptive Fuzzy Logic System (AFLS).

The AFLS was used to adapt the gain and therefore prevent the Kalman filter divergence. The fused signal is more accurate than any of the original signals considered separately. The enhanced, more accurate signal is used to guide and navigate the robot.

3.2 Sensor Fusion

For the navigation system, there are two basic position-estimation methods commonly applied, i.e. relative and absolute positioning, see Borenstein (1996), Shoval, *et al.* (1998), Jetto, *et al.* (1999), Jetto, *et al.* (1999), and Roumeliotis, *et al*. (1999). Relative positioning, which is sometimes called dead reckoning, is usually based on inertial sensors or odometry sensors. In this method, the calculated distance from initial position determines current position estimation. In an absolute positioning system, the positioning sensors interact with a dynamic environment, which can be navigation beacons, active or passive landmark, map matching, or satellite-based navigation signal, to find the position estimation.

To solve the positioning problems, there are two types of sensors available: internal and external sensors, as explained by McKerrow (1991). Internal sensors measure physical variables on the vehicle itself. This self-containing characteristic means the measurement results of these sensors are almost always available to solve positioning problems. The examples of these sensors are accelerometer, odometry, gyroscopes, and compasses. External sensors measure relationships between the vehicle and its environment, which can be natural or artificial objects. The examples of external sensors are satellite signal receiver, sonar sensor, radars, and laser range finders.

When the above sensors are implemented to solve positioning problems, both have advantages and disadvantages. For short periods, measurements using internal sensors are quite accurate. However, for long-term estimation, the measurements usually produce a drift. On the contrary, because it measures absolute quantity, external sensors do not produce the drift, however, the measurements from these sensors are usually not always available, Santini, *et al.* (1997).

The common method used in navigation problem is to combine those sensors so that it will produce the best desirable output. The common combination method is by applying the Extended Kalman Filter (EKF), such as shown in the work by Jetto, *et al.*, (1997, 1999), Tham, *et al.*, (1999), Sasiadek and Wang (1999), Sasiadek and Hartana (2000).

The most common combination of sensors used in positioning and localization problems si combination of odometry and sonar sensor. Odometry sensor is mounted on the robot’s driving wheels and register angular movements of the wheels, which are then translated into linear movements. Beside the drawback that the translation introduces the error, see Sasiadek and Hartana (2000), this implementation makes the odometry signal always available. The sonar sensor, which measures absolute position of the robot, is used to update the position measured by odometer.

Other errors can also occur in odometry sensors. One is systematic error. This error causes the bias in one direction of the movement of the vehicle. Borenstein and Feng (1996) presented their method to correct this error. The method is based on a benchmark experiment performed prior to regular operation of the vehicle. The experiment can find the systematic error and, subsequently, the error is applied to correct the control system parameters. If the systematic errors occur frequently, this method may not be sufficient. For example, if the vehicle uses elastic tires, the benchmarking process has to be performed each time the unequal diameter occurs. It is beneficial that the error correction is done in real time operation.

It is widely known that poorly designed mathematical model for the EKF will lead to the divergence. Clearly, if the plant parameters are subject to perturbations and dynamics of the system are too complex to be characterized by an explicit mathematical model, an adaptive scheme is needed. Jetto, *et al.*, (1999) used Fuzzy Logic Adapted Kalman Filter (FLAKF) to prevent the filter from divergence when fusing measurement from odometry and sonar sensors. In this method, the ratio of innovation and covariance of innovation is used as input to the fuzzy logic, and the output is used to weight the process noise covariance in EKF. Sasiadek and Wang (1999) used exponential data weighting to prevent the divergence. Mean value and covariance of innovation are used as the input of the Fuzzy Logic Adaptive Controller (FLAC). The output is then used to weight process noise and measurement noise covariance in EKF. This FLAC is implemented on the flying vehicle navigating in three-dimensional space. Both those methods have shown improvement in the estimation of the vehicle position in comparison with the EKF only.

In this paper, the systematic error in odometry sensor is corrected during real-time operation of the vehicle by using measurements result from the sonar sensor. EKF is applied to fuse those two signals to find the best estimation of position. Adaptive Fuzzy Logic System (AFLS) is used to prevent the filter from divergence. The objective of this paper is to develop an efficient method for signal fusing to get accurate positioning.

3.3 Mathematical Model

The model of the vehicle used in the simulation is based on a differential-drive. In this model, the vehicle can be steered by differentiating the wheels angular velocity. The kinematic model of this vehicle is described by the following equations, see Wang (1988):

$$\dot{x}(t) = v(t)\sin q(t) \qquad (176)$$
$$\dot{y}(t) = v(t)\sin q(t) \qquad (17)$$
$$\dot{q}(t) = w(t) \qquad (18)$$

where, $v(t)$ and $w(t)$ are, respectively, the linear and angular velocities of the robot, and are expressed by:

$$v(t) = \frac{w_r(t) + w_l(t)}{4} D \qquad (19)$$
$$w(t) = \frac{w_r(t) - w_l(t)}{2d} D \qquad (20)$$

where D and d are the wheel diameter and the distance between the odometry encoder respectively.

If we denote the state variable of the vehicle as $\mathbf{x}(t) = [x(t) \quad y(t) \quad q(t)]^T$, and the vehicle control input as $\mathbf{u}(t) = [v(t) \quad w(t)]^T$, the kinematic model in equations (176) -18) can be written in the form of stochastic differential equation as:

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t), \mathbf{u}(t)) + \mathbf{w}(t) \qquad (21)$$

where $\mathbf{w}(t)$ is a zero-mean Gaussian white noise with covariance matrix $\mathbf{Q}(t)$, which represents the model inaccuracies. This time-equation is linearized and sampled in a small period $T = t_{k+1} - t_k$. Assuming that during this time interval, the linear and angular velocities are constant, and that the vehicle is following an arc path (see Wang (1988)), then, the equations for Extended Kalman Filter can be expressed by:

$$\mathbf{x}_{k+1}^{-} = \mathbf{x}_k + \mathbf{B}_k \mathbf{u}_k \qquad (22)$$
$$\mathbf{P}_{k+1}^{-} = \mathbf{A}_k \mathbf{P}_k \mathbf{A}_k^T + \mathbf{Q}_k \qquad (23)$$
$$\mathbf{K}_{k+1} = \mathbf{P}_{k+1}^{-} \mathbf{C}_{k+1}^T [\mathbf{C}_{k+1} \mathbf{P}_{k+1}^{-} \mathbf{C}_{k+1}^T + \mathbf{R}_{k+1}]^{-1} \qquad (24)$$
$$\mathbf{x}_{k+1} = \mathbf{x}_{k+1}^{-} + \mathbf{K}_{k+1} [\mathbf{y}_{k+1} - \mathbf{C}_{k+1} \mathbf{x}_{k+1}^{-}] \qquad (25)$$
$$\mathbf{P}_{k+1} = [\mathbf{I} - \mathbf{K}_{k+1} \mathbf{C}_{k+1}] \mathbf{P}_{k+1}^{-} \qquad (26)$$

where:

$$\mathbf{x}_k = [x_k \quad y_k \quad q_k]^T \qquad (27)$$

$$\mathbf{B}_k = \begin{bmatrix} T\cos\left(q_k + \frac{\Delta q_k}{2}\right) & 0 \\ T\sin\left(q_k + \frac{\Delta q_k}{2}\right) & 0 \\ 0 & 1 \end{bmatrix} \tag{28}$$

$$\mathbf{A}_k = \begin{bmatrix} 1 & 0 & -v_k T \sin q_k \\ 0 & 1 & v_k T \cos q_k \\ 0 & 0 & 1 \end{bmatrix} \tag{29}$$

$$\mathbf{Q}_k = [\mathbf{Q}_1 \quad \mathbf{Q}_2 \quad \mathbf{Q}_3] \tag{30}$$

$$\mathbf{Q}_1 = \begin{bmatrix} Q_{11}T + Q_{33}\,(T^3/3)v_k^2 \sin^2 q_k \\ -Q_{33}\,(T^3/3)v_k^2 \sin q_k \cos q_k \\ -Q_{33}\,(T^2/2)v_k \sin q_k \end{bmatrix} \tag{31}$$

$$\mathbf{Q}_2 = \begin{bmatrix} -Q_{33}\,(T^3/3)v_k^2 \sin q_k \cos q_k \\ Q_{22}T + Q_{33}\,(T^3/3)v_k^2 \cos^2 q_k \\ Q_{33}\,(T^2/2)v_k \cos q_k \end{bmatrix} \tag{32}$$

$$\mathbf{Q}_3 = \begin{bmatrix} -Q_{33}\,(T^2/2)v_k \sin q_k \\ Q_{33}(T^2/2)v_k \cos q_k \\ Q_{33}T \end{bmatrix} \tag{33}$$

and, $Q_{11} = s_x^2$, $Q_{22} = s_y^2$, and $Q_{33} = s_z^2$ are diagonal elements of covariance matrix $\mathbf{Q}(t)$ of $\mathbf{w}(t)$ in Eq. (21).

The measurement, in this case, will consist of the measurement from odometry sensor and sonar sensor. The size of the measurement vector depends on the number of active sonar sensor. In general, this vector can be expressed as (See Jetto et. al. (1999)):

$$\mathbf{y}(\mathbf{x}_k, \Pi) = [x_k \quad y_k \quad q_k \quad d_{1k} \quad d_{2k} \quad \mathbf{K} \quad d_{nk}]^T \tag{34}$$

where d_{nk} is the measurement of sonar ***n***th at time ***k***.

3.4 Adaptive Fuzzy Logic System

In Kalman filter model, both process noise $\mathbf{w}_k$ and measurement noise $\mathbf{v}_k$ are assumed zero-mean white noise sequence with covariance $\mathbf{Q}_k$ and $\mathbf{R}_k$. If the model of EKF is tuned perfectly, the residual between actual and predicted measurement should be a zero-mean white noise process.

Often, we do not know all parameters of the model or we want to reduce the complexity of modeling. Therefore, in real application, the exact values of $\mathbf{Q}_k$ and $\mathbf{R}_k$ are not known. If the actual process and measurement noises are not a zero-mean white noise, the residual in Kalman filter will also not be a white noise. If this is happened, the Kalman filter would diverge or at best converge to a large bound.

Jetto, *et al*. (1999) used fuzzy logic adapted Kalman filter to prevent the filter from divergence. The fuzzy logic controller uses one input and one output. The ratio between innovation and covariance of innovation process is used as an input. The output is a constant, which is used to weight the process noise covariance. The controller uses five fuzzy rules, and it is implemented in a wheeled mobile robot equipped with odometry and sonar sensors.

Sasiadek and Wang (1999) used fuzzy logic adapted controller (FLAC) to prevent the filter from divergence when fusing signals coming from INS and GPS on flying vehicle. Nine rules were used. There were two inputs, which are the mean value and covariance of innovation, and the output is a constant that is used to weight exponentially the model and measurement noise covariance.

In the case of fusing signals that come from odometry and sonar sensors, sometime only odometry measurements are available. The innovation will be a white noise as long as the process and measurement noises are assumed as a white noise. However, when the sonar measurements become available, and combined with the odometry measurement, the innovation might be not a white noise anymore. This will cause the filter to diverge.

When systematic error occurs in the vehicle, the process and measurement noise actually are not a gaussian white noise, which causes divergence in EKF. AFLS can be used to adapt the filter gain so that the divergence can be avoided. The adaptation process used in this paper is based on exponential data weighting (Lewis, 1986). The scheme of the adaptation process is shown in Fig. 12.

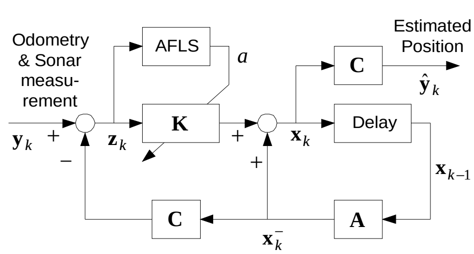


Fig. 12. Adaptive Fuzzy Logic System (AFLS) scheme

The membership function used in this AFLS is displayed in Fig. 13 - Fig.15. The AFLS uses nine rules, which are summarized in Table 1.

3.4.1 Weighted EKF

Using exponential data weighting as an adaptation process, the equation for the EKF will be different. For exponential data weighting, the weighted process and measurement noise covariance can be written as:

$$\mathbf{R}_k = \mathbf{R}a^{-2(k+1)} \tag{35}$$

$$\mathbf{Q}_k = \mathbf{Q}a^{-2(k+1)} \tag{36}$$

where $a \geq 1$. $\mathbf{Q}$ and $\mathbf{R}$ are constant matrices of process and measurement noise covariance. For $a > 1$, as time k increases, $\mathbf{Q}_k$ and $\mathbf{R}_k$ will decrease, which means that the most recent measurement is given higher weighting.

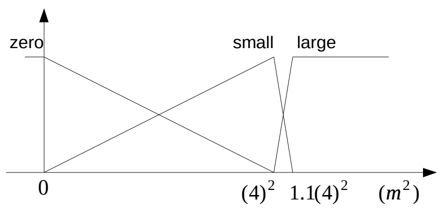


Fig.13. MF of innovation process covariance

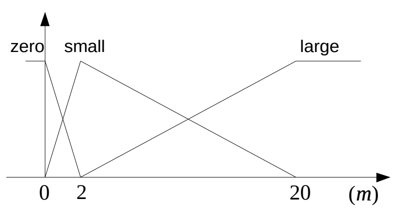


Fig.14. MF of innovation process mean value

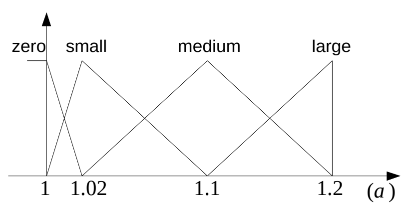


Fig.15. MF of a

Table 7. Rules table for AFLS

a		Innovation process mean value		
		Zero	Small	Large
Innovation process covariance	Zero	Small	Zero	Large
	Small	Zero	Large	Medium
	Large	Large	Medium	Zero

If the weighted estimation covariance is defined as:

$$\mathbf{P}_k^{a-} = \mathbf{P}_k^{-} a^{2k} \quad (37)$$

then the EKF equations become:

$$\mathbf{x}_{k+1}^{-} = \mathbf{x}_k + \mathbf{B}_k \mathbf{u}_k \quad (38)$$

$$\mathbf{P}_{k+1}^{a-} = a^2 \mathbf{A}_k \mathbf{P}_k^{a} \mathbf{A}_k^T + \mathbf{Q}_k \quad (39)$$

$$\mathbf{K}_{k+1} = \mathbf{P}_{k+1}^{a-} \mathbf{C}_{k+1}^T [\mathbf{C}_{k+1} \mathbf{P}_{k+1}^{a-} \mathbf{C}_{k+1}^T + \frac{\mathbf{R}_{k+1}}{a^2}]^{-1} \quad (40)$$

$$\mathbf{x}_{k+1} = \mathbf{x}_{k+1}^{-} + \mathbf{K}_{k+1} [\mathbf{y}_{k+1} - \mathbf{C}_{k+1} \mathbf{x}_{k+1}^{-}] \quad (41)$$

$$\mathbf{P}_{k+1}^{a} = [\mathbf{I} - \mathbf{K}_{k+1} \mathbf{C}_{k+1}] \mathbf{P}_{k+1}^{a-} \quad (42)$$

3.5 Experiments and Results

Simulation experiments have been conducted to show the implementation of AFLS when fusing the signals that come from odometry and sonar sensor. Systematic error in odometry measurement, which comes from unequal in wheel's diameter, is also considered. The vehicle is planned to follow sinus path in in-door environment. The map of the in-door environment along with the movement of the mobile vehicle that has systematic error is shown in Fig. 16.

Three simulation experiments have been performed. The first experiment is to show the implementation of EKF in the mobile robot using odometry sensor, where the sensor has systematic error. The result of this experiment is shown in Fig.17. In this experiment, it shows that the implementation of EKF with only one measurement signal is available, cannot be used to correct the systematic error. The EKF in this case only filters the Gaussian white noise of the odometry measurement error. However, the systematic error is still present in the movement of the mobile vehicle.

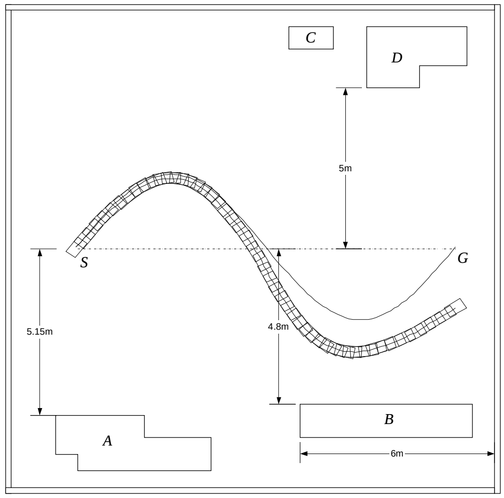


Fig.16. Map of in-door environment

The second experiment is to use the EKF to fuse measurement signals that come from odometry and sonar sensor without using AFLS. This experiment result is shown in Fig. 18. The present of sonar sensor, which measures the relation of the mobile vehicle and its environment, reduces the systematic error, and the mobile vehicle can follow the designed path. However, the movement of the mobile vehicle in this case is not smooth. The result of sonar measurement in this experiment is not used efficiently to improve the position estimation.

The third experiment is to use AFLS to adapt the gain of EKF to prevent the filter from divergence. In this experiment, when the sonar measurement becomes available, the EKF uses this signal to improve its estimation. AFLS makes the position estimation smoother than without AFLS. The result of this experiment is shown in Fig.9.

3.6 Summary

In this chapter, Extended Kalman Filter (EKF) has been used to estimate the position of the mobile vehicle. To prevent the filter from divergence, the innovation and covariance of innovation process are monitored by using Adaptive Fuzzy Logic System (AFLS). The result is an adaptation in the gain of EKF.

Odometry and sonar sensors have been used to illustrate the method. From the simulation experiment, it shows that beside the improvement in the estimation of position, the method can also be used to correct the systematic error. Using this method, real-time operation of the vehicle can be reduced.

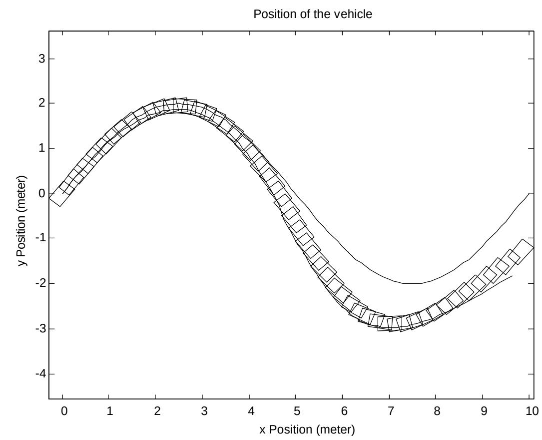


Fig.17. Results of simulation experiment using EKF with only odometry measurement.

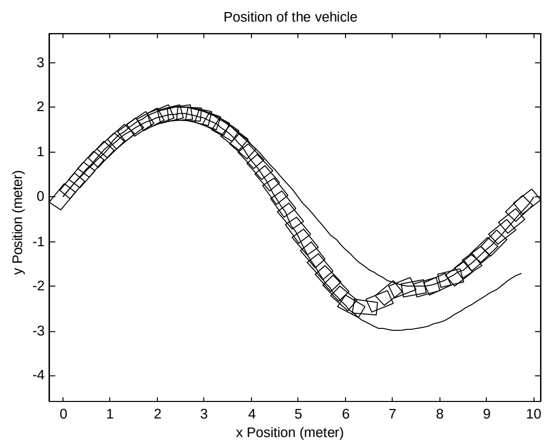


Fig. 18. Simulation experiment result using EKF with odometry and sonar measurement

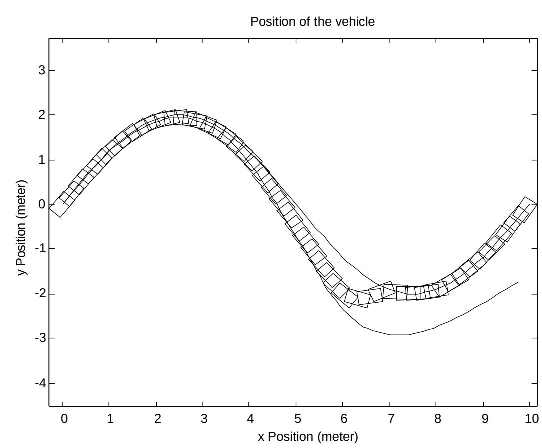


Fig.19. Simulation experiment result using EKF with odometry and sonar measurement, adapted by AFLS

4. Sensor Fusion based on Fuzzy Kalman Filter

4.1 Introduction

In this chapter, a fuzzy Kalman filter was presented, which is based on fuzzy logic theory and Kalman filtering. It is similar to Kalman filter when a linear system with Gaussian noise is considered. However, when non-Gaussian noise is introduced, it is shown that fuzzy Kalman filter is outperforming Kalman filter, while Kalman filter does not work well. It was demonstrated the performance of Kalman filter and fuzzy Kalman filter for position estimation application under different kinds of circumstances. The comparisons are made to draw conclusions.

4.2. Kalman Filter

Experimental measurements are never perfect, even with sophisticated modern instruments. The problem of estimating the state of a stochastic dynamical system from noisy observations taken on the state is of central importance in engineering. Noise filtering is an important part of processing a real signal sequence. There are many kinds of filters could be used for estimation purpose, such as mean filter, median filter, Gaussian filter, and so on. In this article, we discuss the performances of Kalman filter and fuzzy Kalman filter.

There are two basic processes that are modeled by the Kalman filter. The first process is a model describing how the error state vector changes in time. This model is the system dynamics model. The second model defines the relationship between the error state vector and any measurements processed by the filter, and it is the measurement model. Intuitively, the Kalman filter sorts out information and weights the relative contributions of the measurements and of the dynamic behavior of the state vector. The measurements and state vector are weighted by their respective covariance matrices.

The Kalman filter estimates a process by using a form of feedback control: the filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements. As such, the equations for the Kalman filter fall into two groups: time update equations and measurement update equations. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations are responsible for the feedback – i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate.

It was assumed the random process to be estimated can be modeled in the form

$$x_{k+1} = \Phi_k x_k + w_k \qquad (43)$$

The observation (measurement) of the process is assumed to occur as discrete points in time in accordance with the linear relationship

$$z_k = H_k x_k + v_k \qquad (44)$$

The covariance matrices for the w_k and v_k vectors are given by

$$E[w_k w_i^T] = \begin{cases} Q_k, & i = k \\ 0, & i \neq k \end{cases} \qquad (45)$$

$$E[v_k v_i^T] = \begin{cases} R_k, & i = k \\ 0, & i \neq k \end{cases} \qquad (46)$$

$$E[w_k v_i^T] = 0, \quad \text{for all } k \text{ and } i \qquad (47)$$

We also assume that we know the error covariance matrix associated with $\hat{x}_k^-$. That is, we define the estimation error to be

$$e_k^- = x_k - \hat{x}_k^- \qquad (48)$$

and, the associated error covariance matrix is

$$P_k^- = E\left[e_k^- e_i^{-T}\right] = E\left[\left(x_k - \hat{x}_k^-\right)\left(x_k - \hat{x}_k^-\right)^T\right] \qquad (49)$$

It is clear that once the loop is entered, it can be continued ad infinitum. The pertinent equations and the sequence of computation step are shown pictorially in Fig. 20.

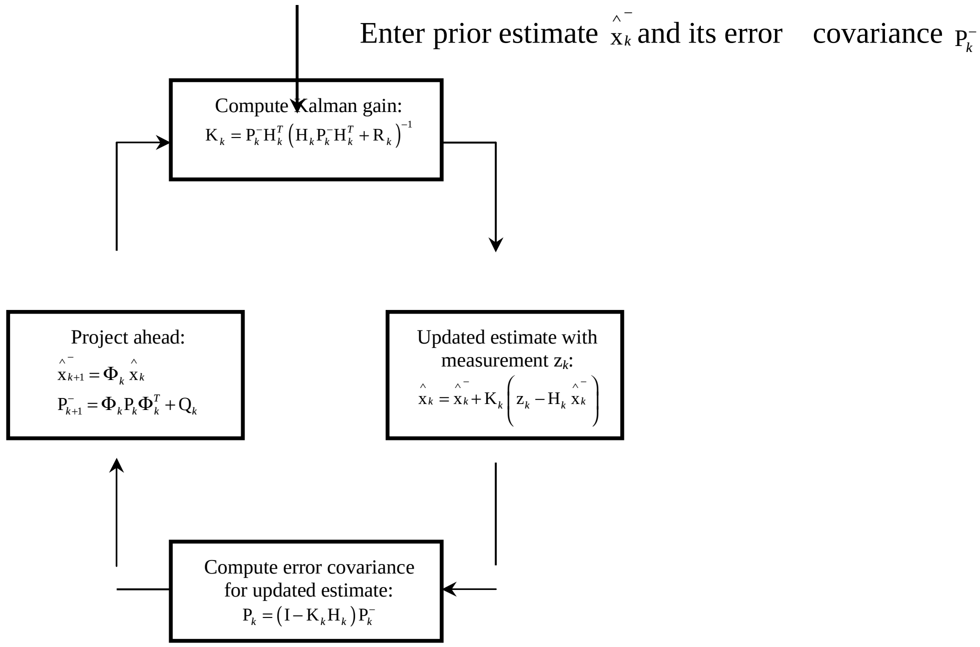


Fig. 20. Kalman Filter Recursive Computation Loop

4.3 Fuzzy Logic Control

Fuzzy logic control is a control method based on fuzzy logic. Just as fuzzy logic can be described simply as “computing with words rather than numbers”; fuzzy logic control can be described simply by “control with sentences rather than equations”.
The basic configuration of the fuzzy logic controller is shown in Fig. 21.

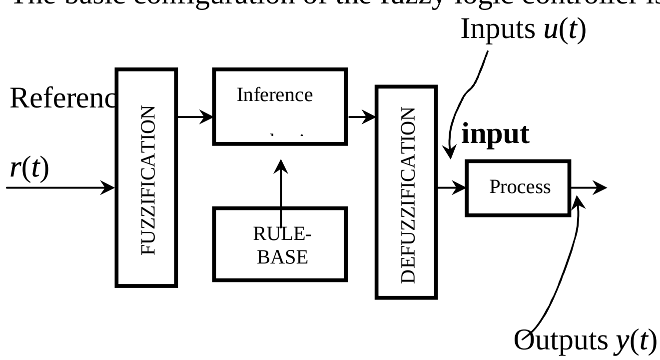


Fig. 21. Fuzzy logic Controller Architecture

1) Rule Base

Specifically, the fuzzy rule-base comprises the following fuzzy If-Then rules:

IF x_1 is A_1^l and ... and x_n is A_n^l, THEN y is B^l

where A_i^l and B^l are fuzzy sets in $U_i \subset R$ and $V \subset R$, respectively, and $\mathrm{x} = (x_1, x_2, ..., x_n)^T \in U$ and $y \in V$ are the input and output (linguistic) variables of the fuzzy system, respectively.

2) Inference Mechanism

The premises of all the rules are compared to the controller inputs to determine which rules apply to the current situation. The "matching" process involves determining the certainty that each rule applies.

3) Fuzzification

The fuzzification process is the act of obtaining a value of an input variable and finding the numeric values of the membership function(s) that are defined for that variable.

4) Defuzzification

Defuzzification operates on the implied fuzzy sets produced by the inference mechanism and combines their effects to provide the "most certain" controller output.

Center of Gravity (COG) method

$$u^{crisp} = \frac{\sum_i b_i \int m_{(i)}}{\sum_i \int m_{(i)}} \quad (50)$$

where

b_i ––center of the membership function of the consequent of rule (i)

$\int m_{(i)}$ ––area under the membership function $m_{(i)}$

Center Average method

$$u^{crisp} = \frac{\sum_i b_i m_{premise_{(i)}}}{\sum_i m_{premise_{(i)}}} \quad (51)$$

4.4 Dynamic System Model

In this chapter a dynamic system model is used which consists of a spacecraft accelerating with random bursts of gas from its reaction control system thrusters, the vector x might consist of position P and velocity V. The dynamic equations are

$$\mathrm{P}_{k+1} = \mathrm{P}_k + \mathrm{V}_k \Delta t + \frac{1}{2} \mathrm{a}_k \Delta t^2 \quad (52)$$

$$\mathrm{V}_{k+1} = \mathrm{V}_k + \mathrm{a}_k \Delta t \quad (53)$$

The system equation is

$$\begin{bmatrix} P_{k+1} \\ V_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_k \\ V_k \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\Delta t^2 \\ \Delta t \end{bmatrix} a_k \qquad (54)$$

where a_k is the random, time-varying acceleration and Δt is the time between step k and step $k+1$. Now suppose we can measure the position P. Then our measurement at time k can be denoted $z_k = P_k + v_k$, where v_k is random measurement noise.
We assume that the process noise w_k is Gaussian noise with a covariance matrix Q. Further assume that the measurement noise v_k is Gaussian noise with a covariance matrix R, and that it is not correlated with the process noise.
The state transition matrix is:

$$\Phi_k = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \qquad (55)$$

The measurement matrix is:

$$H_k = [1 \;\; 0] \qquad (56)$$

Process noise matrix is:

$$w_k = \begin{bmatrix} \frac{1}{2}\Delta t^2 \\ \Delta t \end{bmatrix} * \text{ process noise} \qquad (57)$$

Measurement noise v_k = measurement noise (58)

$$Q_k = E\left[w_k w_k^T \right]$$

$$= E\left[\begin{bmatrix} \frac{1}{2}\Delta t^2 \\ \Delta t \end{bmatrix} * \text{process noise} * \begin{bmatrix} \frac{1}{2}\Delta t^2 & \Delta t \end{bmatrix} * \text{process noise} \right] \qquad (59)$$

$$= \begin{bmatrix} \frac{1}{4}\Delta t^4 & \frac{1}{2}\Delta t^3 \\ \frac{1}{2}\Delta t^3 & \Delta t^2 \end{bmatrix} * \text{process noise}^2$$

$$R_k = E\left[v_k v_k^T \right] \qquad (60)$$

$$= \text{measurement noise}^2$$

The initial parameters chosen for the simulation are:

1) True position trajectory: $x_k = 10*\sin(t_k)$;
2) Standard deviation of position measurement noise: 10 m;
3) Standard deviation of acceleration process noise: 0.5 m/s^2;
4) Total simulation time period: 100 sec.;
5) Time step Δt: 0.2 sec.;

4.5 Simulation Results

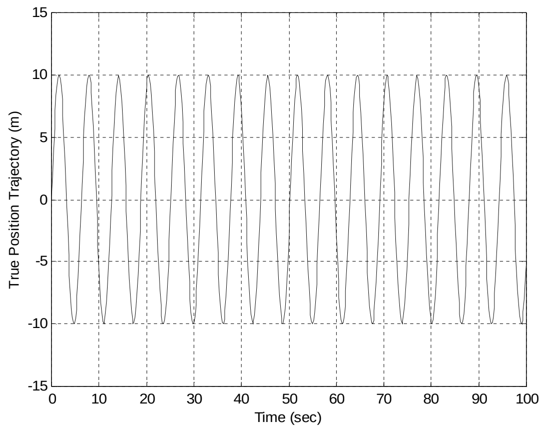


Fig. 22 True Position Trajectory

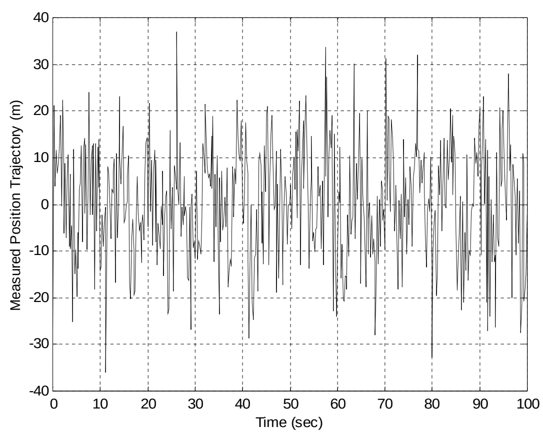


Fig.23 Measured Position Trajectory

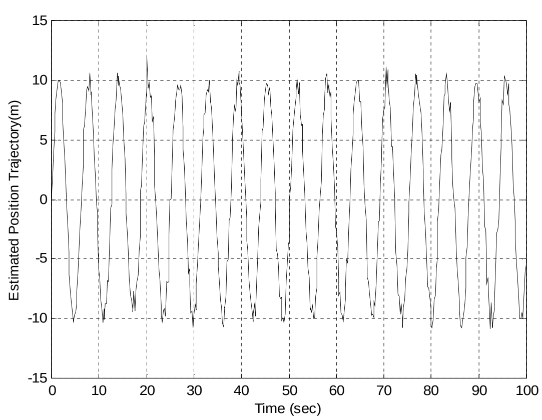


Fig. 24 Estimated Position

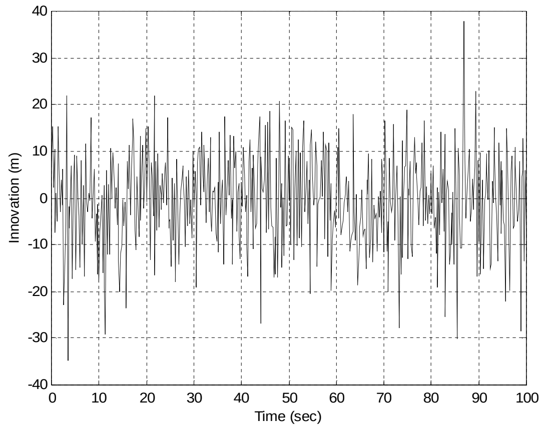


Fig. 25 Innovation

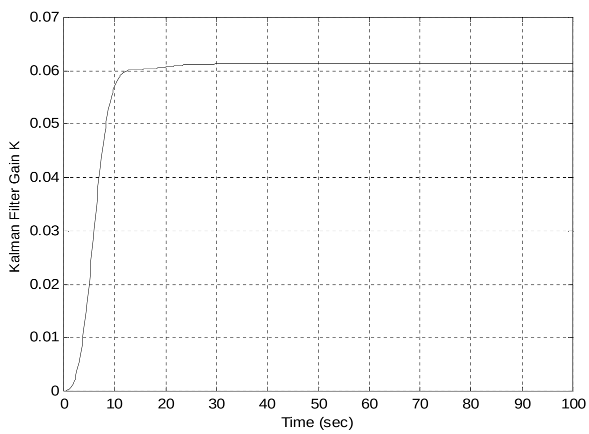


Fig. 26 Kalman Filter Gain

From Fig. 23 to 26, we can see that Kalman Filter works very well in spite of large measurement noise. Kalman gain K converges to a certain value and becomes stable at 0.0613 in this case.

A. Process Noise Covariance Q and Measurement Noise Covariance R [13][14]—Weighted Q and R

$$\begin{aligned} Q_k &= Q a^{-2(k+1)} \\ R_k &= R a^{-2(k+1)} \end{aligned} \qquad (61)$$

where $a > 1$, Q and R are constant matrices.

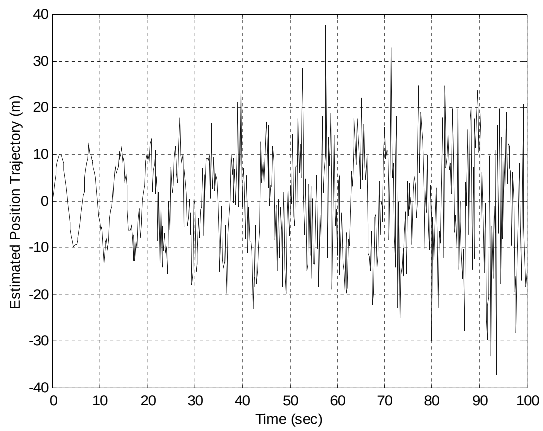


Fig. 27 Estimated Position Trajectory

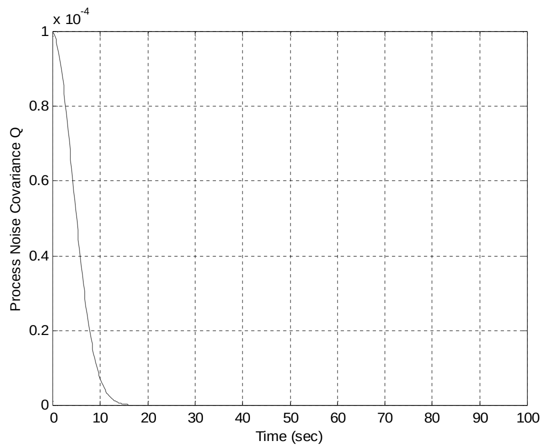


Fig. 28 Process Noise Covariance Q

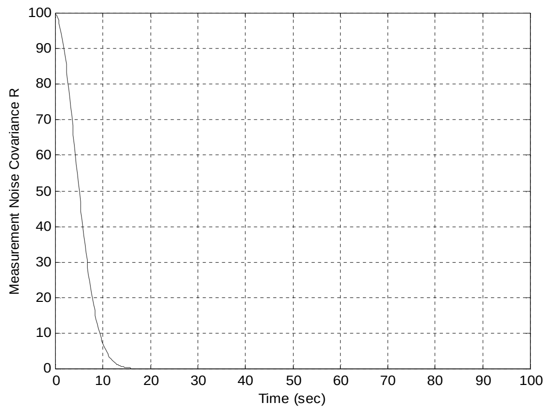


Fig 29 Measured Noise Covariance R

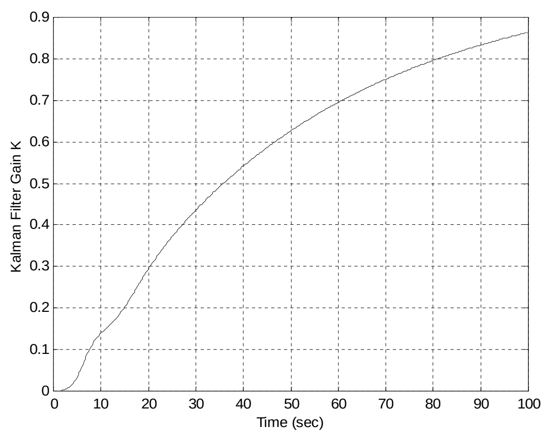


Fig. 30 Kalman Gain K

With the decreasing of Q and R, Kalman gain K becomes diverging and cannot reach a stable value. The estimated position trajectory (Fig. 27) becomes more and more inaccurate too.

B. Correlated Process Noise and Measurement Noise

Let this correlation be described by

$$E\left[\mathrm{w}_k \mathrm{v}_i^T\right] = \begin{cases} \mathrm{C}_k & i = k \\ 0 & i \neq k \end{cases} \tag{62}$$

A generalized derivation yields the optimal estimation algorithm with the same initial conditions and measurement update relations:

$$\hat{x}_{k+1}^{-} = \Phi_k \hat{x}_k + C_k \left(H_k P_k^{-} H_k^T + R_k \right)^{-1} \left(z_k - H_k \hat{x}_k^{-} \right) \qquad (63)$$

$$P_{k+1}^{-} = \Phi_k P_k \Phi_k^T + Q_k - C_k \left(H_k P_k^{-} H_k^T + R_k \right)^{-1} C_k^T \qquad (64)$$
$$- \Phi_k K_k C_k^T - C_k K_k^T \Phi_k^T$$

$$C_k = E\left[w_k v_k^T \right] = \begin{bmatrix} \frac{1}{2}\Delta t^2 \\ \Delta t \end{bmatrix} * \text{ process noise}$$
$$* \text{ measurement noise} \qquad (65)$$

This can be compared to the case of no correlation between w_k and v_k. The decrease in steady state values from

$P_k^- = \begin{bmatrix} 6.5286 & 1.0321 \\ 1.0321 & 0.3213 \end{bmatrix}$ to $\begin{bmatrix} 0.0353 & 0.0172 \\ 0.0172 & 0.0084 \end{bmatrix}$ and $P_k = \begin{bmatrix} 6.1285 & 0.9689 \\ 0.9689 & 0.3113 \end{bmatrix}$ to $\begin{bmatrix} 0.0287 & 0.0158 \\ 0.0158 & 0.0087 \end{bmatrix}$

is due to the exploitation of the correlation between the dynamic noise and the noise that corrupts the observable outputs: the z_k realizations reveal more about the noise process w_k.

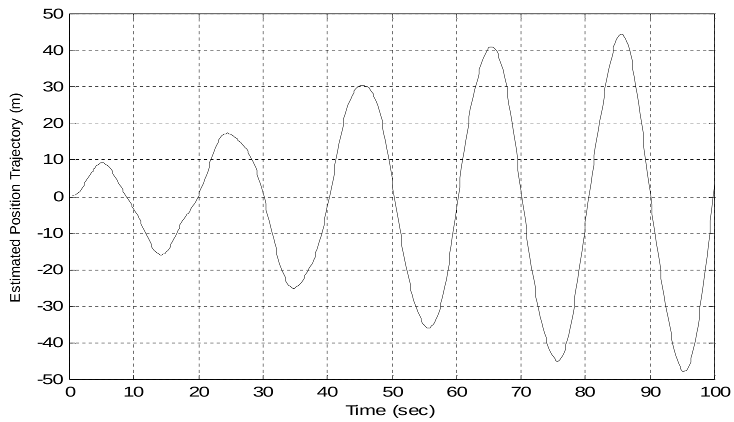


Fig. 31. Estimated Position

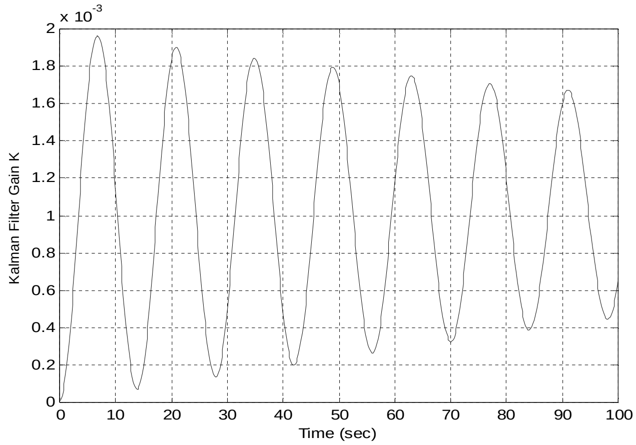


Fig. 32. Kalman Gain K

4.6 Fuzzy Kalman Filter

Now we introduce an "optimal" state estimator, based on fuzzy set theory, which is capable of dealing with systems with random disturbances and "uncertainty". We will refer to this as a fuzzy Kalman filter, and it is a fuzzy system model of the process in the estimator. This filter is similar to that of the Kalman filter when a linear system with Gaussian noise is considered.

Let the inputs be $e_k, \dot{e}_k, e_{P_k}, \dot{e}_{P_k}$, and the output is the estimated position at time step k+1, where e_k is the error between measured position value and true position value, $\dot{e}_k$ is the change in e_k, e_{P_k} is the difference between P_k and P_k^-, and $\dot{e}_{P_k}$ is the change in e_{P_k}. Here we use a MISO (Multiple Inputs Single Output) system to accomplish the task. The rules being used are such as

Rule 1: IF e_k is negative large, $\dot{e}_k$ is zero, e_{P_k} is negative large, and $\dot{e}_{P_k}$ is negative large, THEN estimated position at step k+1 is positive large.

Rule 2: IF e_k is zero, $\dot{e}_k$ is zero, e_{P_k} is negative large, and $\dot{e}_{P_k}$ is negative large, THEN estimated position at step k+1 is positive large.

Rule 3: IF e_k is positive large, $\dot{e}_k$ is zero, e_{P_k} is zero, and $\dot{e}_{P_k}$ is zero, THEN estimated position at step k+1 is negative large.

Rule 4: IF e_k is zero, $\dot{e}_k$ is positive small, e_{P_k} is negative small, and $\dot{e}_{P_k}$ is negative small, THEN estimated position at step k+1 is negative small.

The results are plotted as in Fig. 33 to Fig. 36. Compared with Fig. 24 which is obtained by using Kalman filter, it is more accurate.

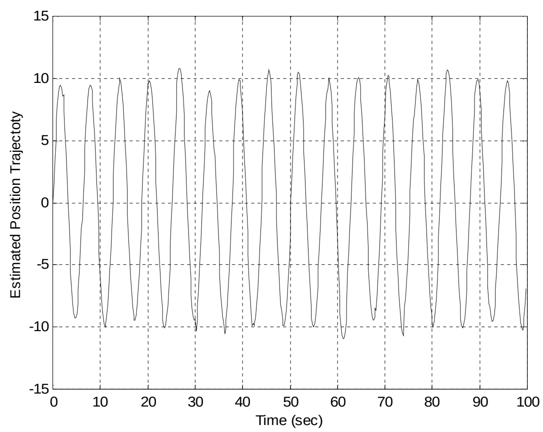


Fig. 33 Estimated Position

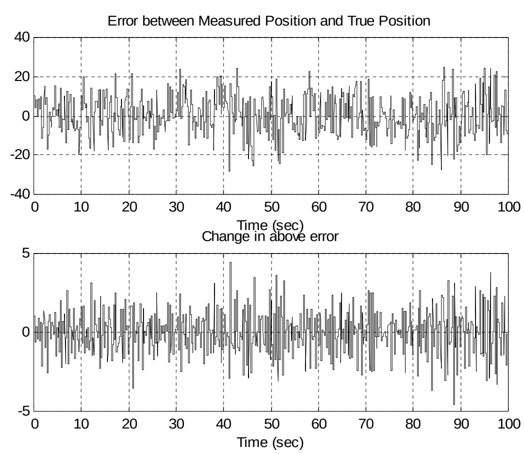


Fig. 34 e_k and $\dot{e}_k$

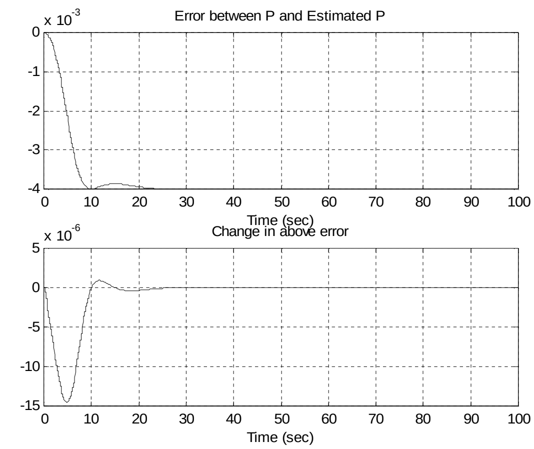


Fig. 35 e_{P_k} and $\dot{e}_{P_k}$

A. Process Noise Covariance Q and Measurement Noise Covariance R—Weighted Q and R (as in Kalman filter)

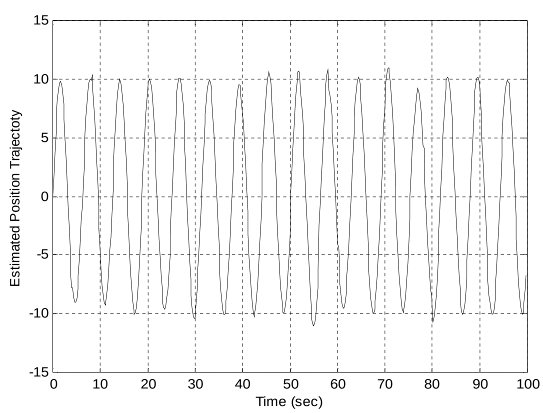


Fig. 36 Estimated Trajectory

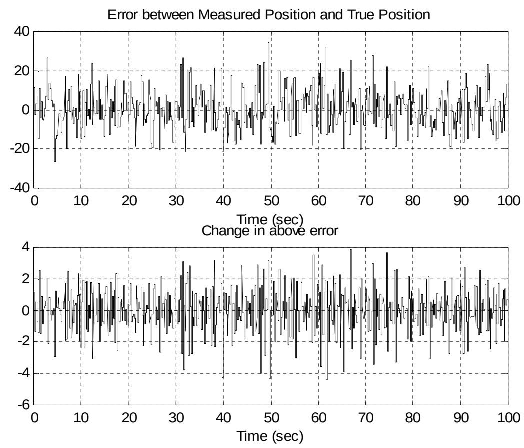
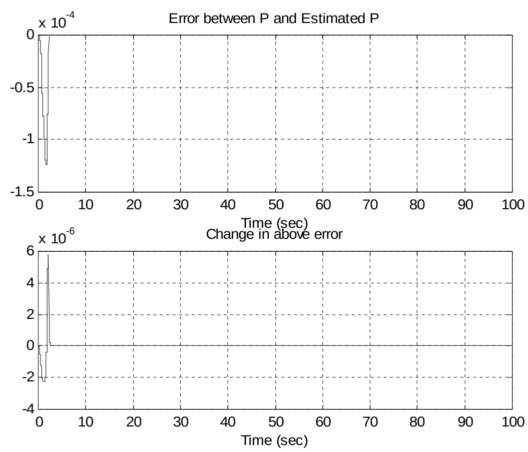


Fig. 37 e_k and $\dot{e}_k$



B. Correlated Process Noise and Measurement Noise (as in Kalman filter)

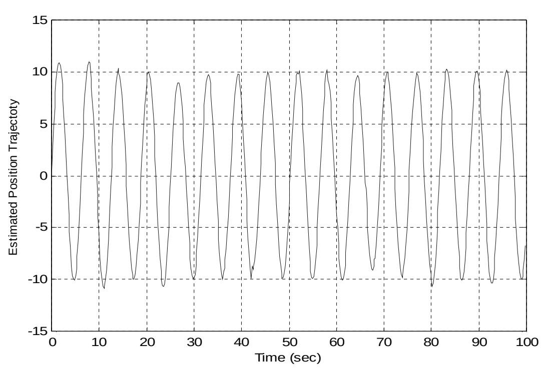


Fig. 39 Estimated Position

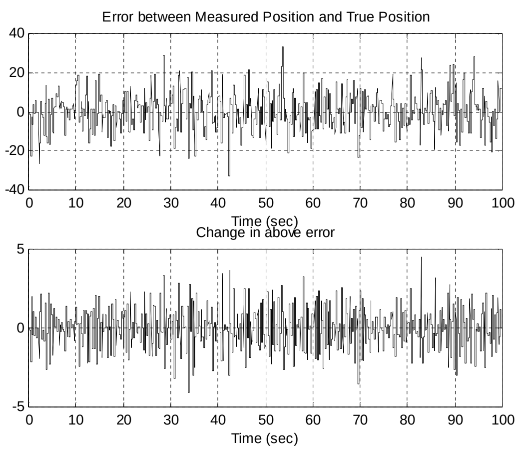


Fig. 40 e_k and $\dot{e}_k$

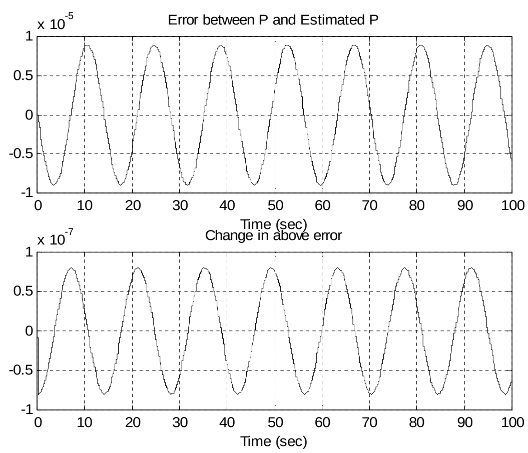


Fig. 41 e_{P_k} and $\dot{e}_{P_k}$

4.7 Summary

The method that is the most widely used for sensor fusion in engineering applications is the Kalman filter. This filter is often used to combine all measurement data (e.g., for fusing data from different sensors) to get an optimal estimate in a statistical sense. If the system can be described with a linear model and both the system error and the sensor error can be modeled as Gaussian noise, then the Kalman filter will provide a unique statistically optimal estimate for the fused data. This means that under certain conditions the Kalman filter is able to find the best estimates based on the "correctness" of each individual measurement. On the basis of simulation performed in this chapter for fuzzy Kalman filter and regular Kalman filter under different conditions and parameters, it was shown that for linear systems and triangular shaped membership functions, the fuzzy Kalman filter works similarly as Kalman filter but produces better results than the Kalman filter. In Fig. 42 and 43, the general comparison of fuzzy Kalman Filter and regular Kalman Filter was presented. It can be proved that the convergence of Kalman Filter's estimate to the true value is guaranteed only when the system is stochastically controlled and observed.

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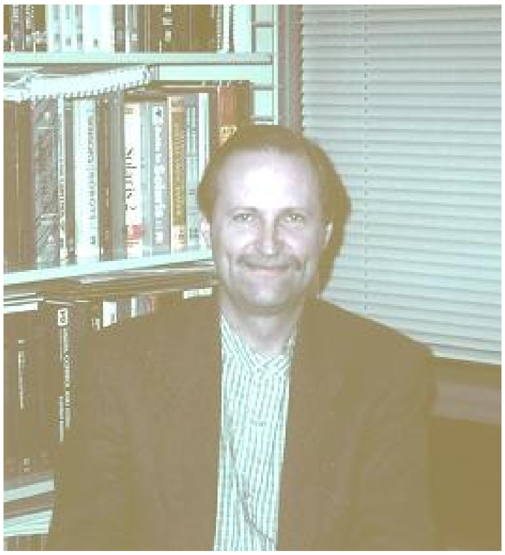

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